Wave Optics
Interference

"Let there be electricity and magnetism; and there is light."

1 Electromagnetic Waves

Had Maxwell said the above quoted opening statement, there would hardly be any doubt about its truthfulness. By the end of 19th century it was well established that light is but a oscillation of the electric and magnetic fields in space that travels as time goes by. The following four equations (**Maxwell's equations**):

$$\text{(i)} \quad \oint \epsilon_0 \vec{E} \cdot d\vec{A} = q \qquad \text{Gauss's Law} \tag{1}$$

$$\text{(ii)} \quad \oint \vec{B} \cdot d\vec{A} = 0 \tag{2}$$

$$\text{(iii)} \quad \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \text{Faraday's Law of Electromagnetic Induction} \tag{3}$$

$$\text{(iv)} \quad \oint \vec{B} \cdot d\vec{l} = \mu_0 i + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \qquad \text{Ampere's Law (Modified by Maxwell)} \tag{4}$$

clearly show that light is nothing but an *electromagnetic wave*. In the following, we describe the basic properties of electromagnetic waves.

1.1 Basic Properties of Electromagnetic Waves

We describe the simplest possible solution of the Maxwell's wave equation: a *plane monochromatic wave*. As the name suggests, in this kind of wave, only a single frequency is present.

It turns out that, if the wave itself travels along the $+z$ direction, the electric and magnetic field vectors are:

$$\vec{E}(x,y,z,t) = E_0 \cos(kz - \omega t + \varphi_0)\ \hat{i} \tag{5}$$

$$\vec{B}(x,y,z,t) = B_0 \cos(kz - \omega t + \varphi_0)\ \hat{j} \tag{6}$$

In these equations, E_0 and B_0 are the amplitudes (i.e., maximum values) of the electric field and magnetic fields respectively. Unit vectors $\hat{i}$ and $\hat{j}$ represent directions along the x and y axis respectively. Note that the electric and magnetic fields' oscillations are in *phase* and that the electric field at all times is perpendicular to the magnetic field and both are perpendicular to the direction of propagation which the positive z direction in this case. At time $t = 0$, the phase at the origin $(x = y = z = 0)$ is φ_0.

Also, the following hold:

$$k = \frac{2\pi}{\lambda}; \qquad \omega = \frac{2\pi}{T} = 2\pi\nu = kc, \tag{7}$$

where $c = 3.0 \times 10^8$ is the speed of light in vacuum and is a universal constant.

Further, it can be proved that

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}, \tag{8}$$

and at any instant of time the following relation holds between the strength of electric and magnetic fields in an em wave:

$$E = cB. \tag{9}$$

Finally, Figure 1 shows a "snapshot" of the em wave taken at a certain moment of time.

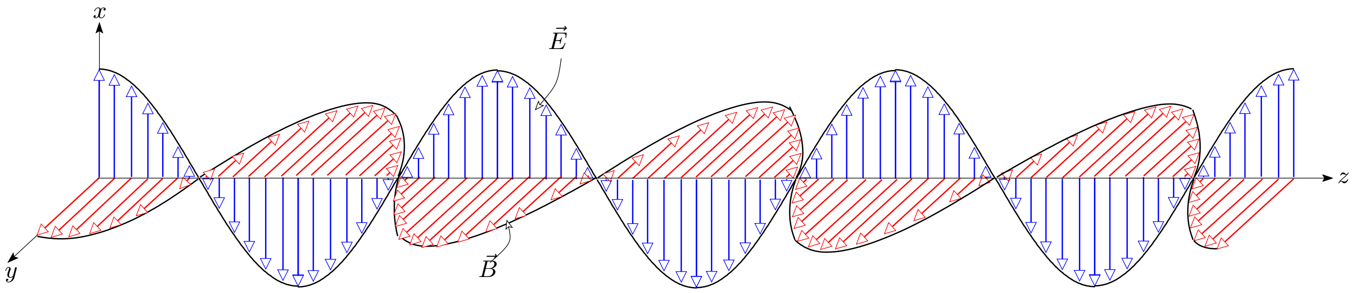

Figure 1: The variation of the electric and magnetic field vectors as a function of z at any fixed instant of time (take a "snapshot"). Here φ_0 is zero. Note that E and B oscillations are in the same phase.

1.1.1 Intensity in Electromagnetic Waves

▷ **Definition 1** (**Intensity**). The intensity of any wave is defined as the power flowing across an area (perpendicular to the direction of propagation of the wave) per unit area.

The energy density of the electromagnetic wave is

$$u = u_E + u_B = \frac{1}{2}\ \epsilon_0 E^2 + \frac{1}{2\mu_0}\ B^2.$$

The energy enclosed in the cuboid in Figure of volume $A\ dz$ is thus,

$$dU = u(A\ dz) = \left(\frac{1}{2}\ \epsilon_0 E^2 + \frac{1}{2\mu_0}\ B^2\right)\ Adz.$$

This energy will cross the cross–section area A in time $dt = dz/c$ where c is the speed of light. Thus intensity I is

$$I = \frac{dU}{A\ dt} = \frac{1}{A\ (dz/c)}\left[\left(\frac{1}{2}\ \epsilon_0 E^2 + \frac{1}{2\mu_0}\ B^2\right)\ Adz\right].$$

Hence, the intensity is

$$I = \left(\frac{1}{2}\ \epsilon_0 E^2 + \frac{1}{2\mu_0}\ B^2\right)\ c. \tag{10}$$

Since, the electric and magnetic field at any point of space oscillate harmonically, in general, the intensity at any point fluctuates harmonically too. But the *frequency of oscillation of the fields is very high* (of the order of 10^{14} Hz), therefore, any instrument (or the human eye) respond only to the **average value** (averaged over millions of cycles) of the intensity. Thus the average value of the intensity is the practically important (and useful) quantity. Since the oscillations of the fields are harmonic (cosine), the *average of the square* over one (or many) cycles is 1/2. Hence,

$$\langle I \rangle = \left(\frac{1}{4}\ \epsilon_0 E_0^2 + \frac{1}{4\mu_0}\ B_0^2\right)\ c, \tag{11}$$

where E_0 and B_0 are the amplitudes of the electric and magnetic fields respectively. Since for electromagnetic field, the relation

$$E_0 = cB_0$$

holds, we can write the Eq. 11 can be written as

$$\langle I \rangle = \frac{1}{2} c\epsilon_0 E_0^2 = \frac{1}{2\mu_0} E_0 B_0. \tag{12}$$

Thus, in em waves *the average intensity is proportional to the square of the amplitude of the electric field.*

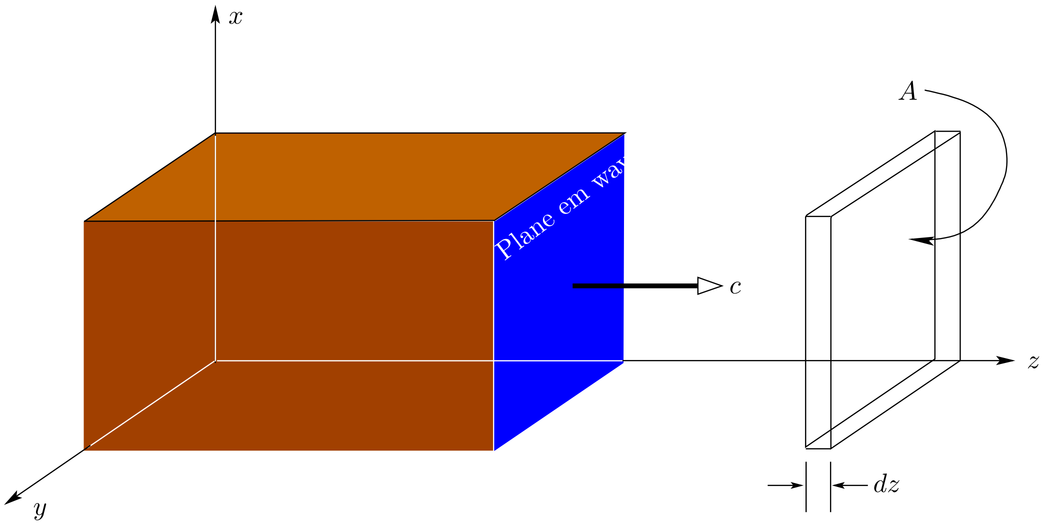

Figure 2: The intensity of an electromagnetic wave.

1.2 Rays and Wavefronts

The *phase* of the wave is the argument of the cosine in the equation (5) or (6). We have the following definition:

▷ **Definition 2 (Wavefront).** The locus of points having the same phase is known as a wavefront.

Using this definition, if we set

$$kz - \omega t + \varphi_0 = \text{ const}, \tag{13}$$

we get that for a *fixed instant of time*, $z =$ some constant, which is the equation for a plane parallel to xy plane. Thus, we see why the wave defined by (5) and (6) are called a plane wave. The speed with which the wavefronts move away from the source is known as the *phase* velocity. Differentiating Equation 13 with respect to time, we get

$$\frac{dz}{dt} = \frac{\omega}{k} = c \quad \Rightarrow v_p = c.$$

Thus for a plane monochromatic electromagnetic wave, the *phase velocity is the same as the wave velocity.*

▷ **Definition 3 (Ray).** Normals drawn to the wavefronts in the direction of *increasing* phase are known as *rays.*

It turns out that *the energy of the wave moves along the rays.* An important general principle is that *the time taken for light to travel from one wavefront to another is the same along any ray.*

2 Huygens' Construction

According to Huygens *each point on a wavefront acts like a source of secondary spherical wavefronts.* Using this principle, he explained the propagation of em wave. The following illustrates the working in the simple case of a plane wave.

(a) At time $t = 0$, draw a surface F_1 which we call a *front*. It separates those parts of the medium which are undisturbed from where the wave has already reached.

(b) Each point on F_1 acts as a source and sends out a spherical wavefront of radius vt. This is called the secondary wavefront.

(c) After a time t, the disturbance would have reached all points within the region covered by all these secondary wavefronts. the boundary of this region is the new front F_2. Note that F_2 is a surface tangent to all the spheres. It is called the *forward envelope* of these secondary wavefronts.

(d) The construction can be repeated starting with F_2 to get the next wavefront F_3 a time t later, and so on.

The above construction is depicted in Figure 3.

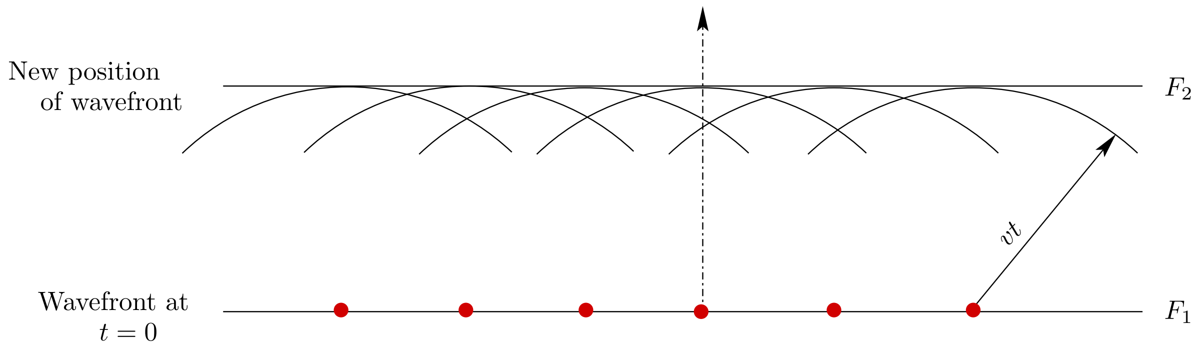

Figure 3: Huygens' construction for wave propagation.

2.1 Reflection and Refraction

Based on the construction of Huygens, reflection of light from a surface and its refraction when passing through an interface between two media can be understood easily. (See Figure 6 for reflection and Figure 7 for refraction.) Consider Figure 4 which shows the incident and reflected wavefronts when a parallel beam of light falls on a plane surface. One ray POQ is shown normal to both the incident and reflected wavefronts. The angles of incidence i and reflection r are defined as the angles made by the incident and reflected rays with the normal. As shown in Figure 4, these are also the angles between the wavefront and the surface. We now calculate the total time the ray takes to go from P to Q in Figure 4. Total time

$$\begin{aligned} t &= \frac{PO}{v} + \frac{OQ}{v} = \frac{AO\sin i}{v} + \frac{OB\sin r}{v} \\ &= \frac{OA\sin i + (AB - OA)\sin r}{v} = \frac{AB\sin r + OA(\sin i - \sin r)}{v}. \end{aligned}$$

Different rays normal to the incident wavefront strike the surface at different points O and hence have different values of OA. Since the time should be the same for all these rays, the last expression

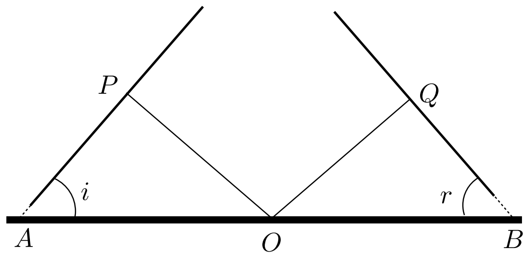

Figure 4: Wavefronts and corresponding rays for reflection from a plane surface.

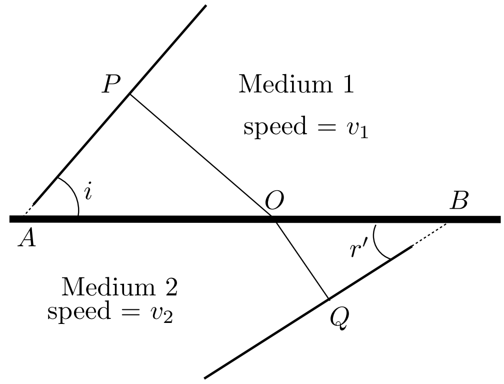

Figure 5: Wavefronts and corresponding rays for refraction at a plane interface.

must be independent of OA. The required criteria for this to happen is that the coefficient of OA must be zero, i.e. $\sin i = \sin r$. We thus have the law of reflection

$$i = r. \tag{14}$$

We now come to refraction. Figure 5 shows a plane surface separating medium 1 (speed of light v_1) from medium 2 (speed of light v_2). The incident and refracted wavefronts making angles i and r' with the normal at the point of incidence are also shown. The angle r' is called the angle of refraction. Rays perpendicular to these wavefronts are also drawn. As before, let us calculate the time taken to travel between two wavefronts along any ray. Time taken from P to Q is

$$\begin{aligned} t &= \frac{PO}{v_1} + \frac{OQ}{v_2} = \frac{OA \sin i}{v_1} + \frac{(AB - OA)\sin r'}{v_2} \\ &= \frac{AB}{v_2}\sin r' + OA\left(\frac{\sin i}{v_1} - \frac{\sin r'}{v_2}\right) \end{aligned}$$

This time again should be independent of the length OA and hence we must have

$$\frac{\sin i}{\sin r'} = \frac{v_1}{v_2}. \tag{15}$$

This is Snell's law of refraction. The ratio of the phase velocity of light c in vacuum to its value v_1 in a medium is called the *refractive index* n_1 of that medium:

$$n_1 = \frac{c}{v_1}. \tag{16}$$

Thus (15) can be written as

$$\frac{\sin i}{\sin r'} = \frac{n_1}{n_2} \quad \Rightarrow \quad n_1 \sin i = n_2 \sin r'. \tag{17}$$

Also since the speed of light in any medium is related to the permittivity and permeability of that medium, the refractive index of any medium can be related to these. This relation is:

$$n = \sqrt{\epsilon_r \mu_r}, \tag{18}$$

where ϵ_r is the *relative permittivity* (dielectric constant) and μ_r is the *relative permeability* of that medium *measured at the frequency of light for which the refractive index is required.*

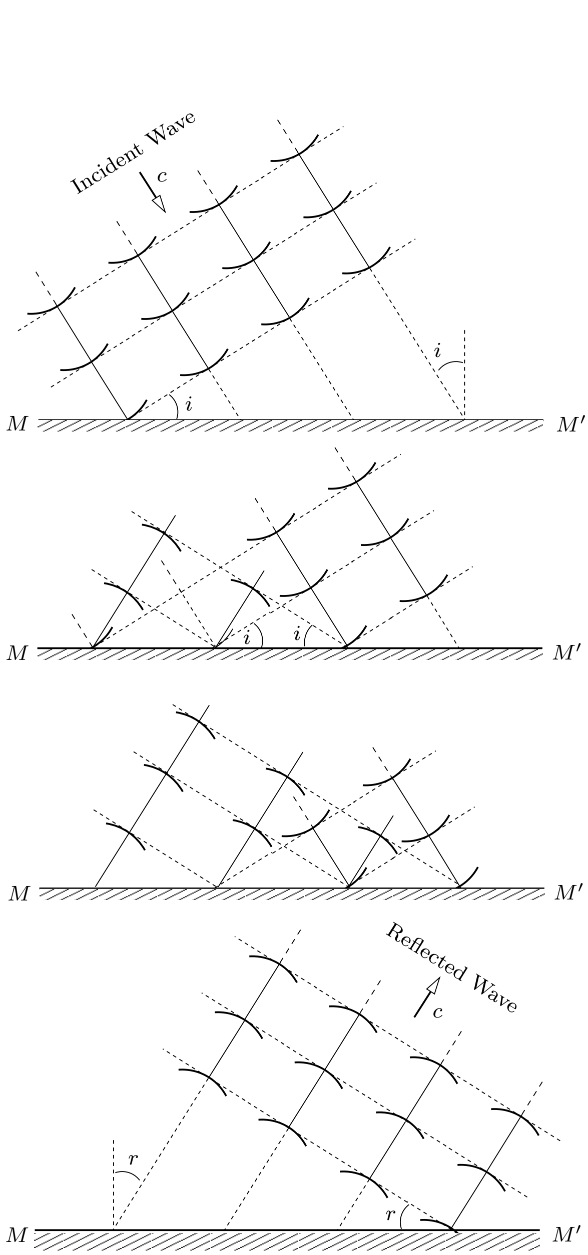

Figure 6: Reflection of light explained by Huygens' construction.

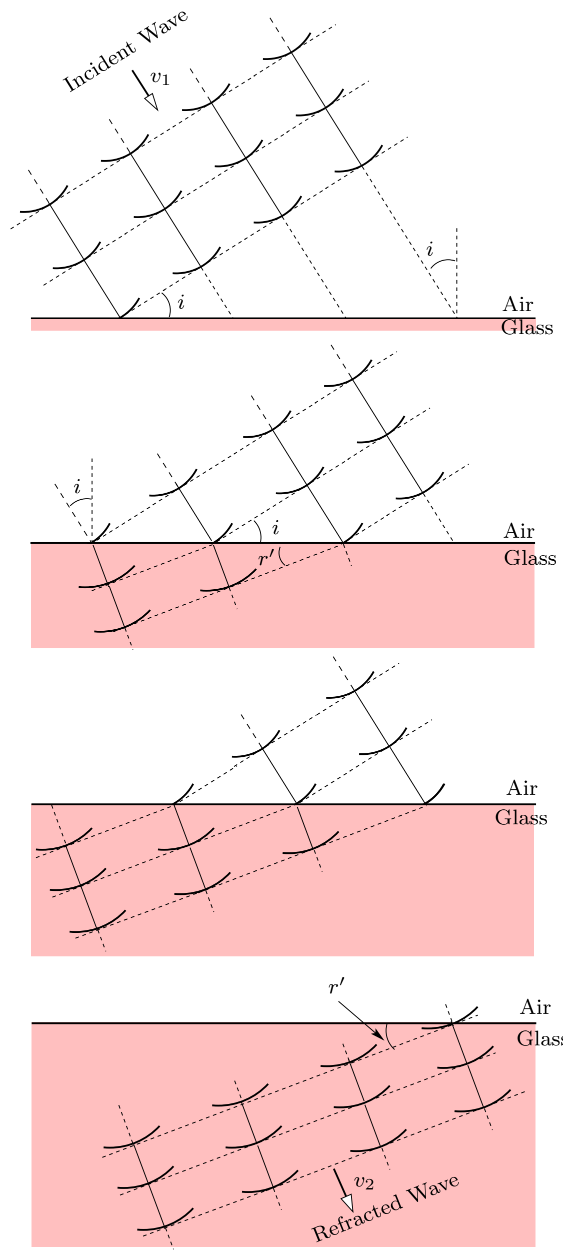

Figure 7: Huygens' construction for refraction of light at an interface.

3 Interference

Our eyes (or any optical instrument) responds to an electromagnetic wave by the interaction of the atomic charges (electrons) with the electric and magnetic fields present in the wave. If F_e denotes the electric force and F_m the magnetic force on a charged particle interacting with the em wave, then we have

$$\frac{F_m}{F_e} = \frac{qvB}{qE} = \frac{vB}{E} = \frac{B}{E}v = \frac{v}{c}.$$

Here v is the velocity of the charged particle (which will move once it interacts with the electric field component of em wave). Since in normal situations $v << c$, we see that the magnetic force is much smaller than the electric force. Thus, any optical device interacts only very weakly with the magnetic field component of the em wave.

Thus, for all practical purposes, we can just concentrate on the electric field of the wave, though the magnetic field is always present.

3.1 Coherence

For electromagnetic waves, the electric fields produced by various source add vectorially (superposition principle). The intensity I at a given point of space is, as we have seen already, proportional to the *square* of the electric field $\vec{E}$, i.e. the dot product of $\vec{E}$ with itself.

$$I = k\vec{E} \cdot \vec{E},$$

where k is the constant of proportionality.

We shall now see that the intensities due to two sources *do not add in general*. Let $\vec{E}_1$ and $\vec{E}_2$ be the electric fields produced *at* a point *by* two point sources 1 and 2, and I_1 and I_2 be their intensities when *acting alone*. Let $\vec{E}$ and I be the electric field and intensity when both are present. We then have:

$$I_1 = k\vec{E}_1 \cdot \vec{E}_1, \qquad I_2 = k\vec{E}_2 \cdot \vec{E}_2, \qquad \vec{E} = \vec{E}_1 + \vec{E}_2,$$

and so

$$\begin{aligned} I &= k(\vec{E}_1 + \vec{E}_2) \cdot (\vec{E}_1 + \vec{E}_2) \\ &= k\vec{E}_1 \cdot \vec{E}_1 + k\vec{E}_2 \cdot \vec{E}_2 + 2k\vec{E}_1 \cdot \vec{E}_2 \\ &= I_1 + I_2 + 2k\vec{E}_1 \cdot \vec{E}_2 \end{aligned} \tag{19}$$

Equation 19 shows that the intensity observed I equals the sum of the individual intensities $I_1 + I_2$ plus one more term

$$I_{12} = 2k\vec{E}_1 \cdot \vec{E}_2. \tag{20}$$

This third term is called the *interference term.* Being a dot product of two vectors, it can be positive, negative, or zero.

The calculation so far was given for an instant of time. But as I have already said, the field vectors in an em wave oscillate about 10^{-15} times in one second, what is practically important is the averaged values over a large number of cycles. And so we need to take the average value of the intensity. Let us do so for the special case when the two em waves are moving parallel to each other. In this case the fields $\vec{E}_1$ and $\vec{E}_2$ are also parallel (or antiparallel) to each other and at any point in space they are oscillating harmonically. Let us take them along the direction $\hat{n}$ as

$$\vec{E}_1 = \hat{n}\ E_{10}\cos(\omega_1 t + \varphi_1), \qquad \text{and} \qquad \vec{E}_2 = \hat{n}\ E_{20}\cos(\omega_2 t + \varphi_2),$$

where E_{10} and E_{20} are, respectively, the amplitudes of the two oscillating fields. Then, we have

$$I_1 = kE_{10}^2\cos^2(\omega_1 t + \varphi_1) \quad \Rightarrow \quad \langle I_1\rangle = kE_{10}^2\langle\cos^2(\omega_1 t + \varphi_1)\rangle = \frac{1}{2}\ kE_{10}^2,$$

$$\text{and} \quad I_2 = kE_{20}^2\cos^2(\omega_2 t + \varphi_2) \quad \Rightarrow \quad \langle I_2\rangle = kE_{12}^2\langle\cos^2(\omega_2 t + \varphi_2)\rangle = \frac{1}{2}\ kE_{20}^2.$$

Now, we take the interference term:

$$\begin{aligned} I_{12} &= 2k\hat{n}\cdot\hat{n}E_{10}E_{20}\cos(\omega_1 t + \varphi_1)\cos(\omega_2 t + \varphi_2) \\ &= kE_{10}E_{20}\left\{\cos[(\omega_1 + \omega_2)t + \varphi_1 + \varphi_2] + \cos[(\omega_1 - \omega_2)t + (\varphi_1 - \varphi_2)]\right\} \end{aligned} \tag{21}$$

Now if we take the time average of the above expression, the first term inside the curly bracket averages to zero. For the remaining term, we obtain

$$\langle I_{12}\rangle = kE_{10}E_{20}\langle\cos[(\omega_1 - \omega_2)t + (\varphi_1 - \varphi_2)]\rangle.$$

We consider two cases:

(a) $\omega_1 \neq \omega_2$: In this case, we have

$$\langle \cos[(\omega_1 - \omega_2)t + (\varphi_1 - \varphi_2)] \rangle = 0.$$

Thus, *the average intensities of two em waves with different frequencies add.*

(b) $\omega_1 = \omega_2$: In this case, we have

$$\langle \cos[(\omega_1 - \omega_2)t + (\varphi_1 - \varphi_2)] \rangle = \langle \cos(\varphi_1 - \varphi_2) \rangle.$$

Let us introduce the notion

$$\delta = \varphi_1 - \varphi_2. \tag{22}$$

Then we have

$$\langle I_{12} \rangle = kE_{10}E_{20} \ \cos\delta. \tag{23}$$

Therefore, the *interference term is proportional to the cosine of the phase difference between the two interfering waves*. However, when we speak of two different sources radiating at the same frequency, the motion of the charges (which is what is responsible for em wave production) are independent. Therefore, the phase difference between them δ varies randomly over the entire range of 0 to 2π. And, therefore, in this case too we get $I_{12} = 0$. Thus, we conclude that *The average intensities from two* ***independent*** *sources, even of the same frequency, add.* Such **sources for which the phase difference does not remain stable (or fixed) are called *incoherent*.**

To see the interference term in action, we, thus, must meet two requirements on the part of the interfering waves:

- they must have the same frequency; and
- the phase difference between them must be stable.

Sources with the same frequency and stable phase difference are called *coherent*.

If these requirements are met, we obtain the total average intensity as:

$$\langle I \rangle = \langle I_1 \rangle + \langle I_2 \rangle + kE_{10}E_{20} \ \cos\delta = \langle I_1 \rangle + \langle I_2 \rangle + 2\sqrt{\langle I_1 \rangle}\sqrt{\langle I_2 \rangle} \ \cos\delta. \tag{24}$$

The *average intensity is maximum, when the two waves are in phase* ($\delta = 0$) :

$$\langle I \rangle_{max} = \left(\sqrt{\langle I_1 \rangle} + \sqrt{\langle I_2 \rangle}\right)^2. \tag{25}$$

In this case, the two waves are said to interfere ***constructively***. The *average intensity is minimum when the two waves are out of phase by* 180° ($\delta = \pi$) :

$$\langle I \rangle_{min} = \left(\sqrt{\langle I_1 \rangle} - \sqrt{\langle I_2 \rangle}\right)^2. \tag{26}$$

This phenomena is termed ***destructive*** interference. As a special case, we note that if the amplitudes of both the waves be equal:

$$E_{10} = E_{20} = E_0, \quad \Rightarrow \quad \langle I_1 \rangle = \langle I_2 \rangle = \frac{1}{2}k \ E_0^2 = I_0,$$

we get the average intensity as

$$\langle I \rangle = 2I_0 \left[1 + \cos\delta\right] \tag{27}$$

and thus the minimum intensity observed in this case is zero (there is darkness even when two light sources are there!) and the maximum intensity is $4I_0$!

3.2 Young's Experiment

An experiment first carried out by the British scientist Thomas Young and commonly called the Young's double slit experiment uses the concept of interference to demonstrate the wave nature of light and at the same time can be used to measure the wavelength of light. The experimental set up (as well as the result, which we shall study shortly) is shown in its elementary form in Figure 8(a). Two slits S_1 and S_2 are made in an opaque screen, parallel to each other and very close. These two

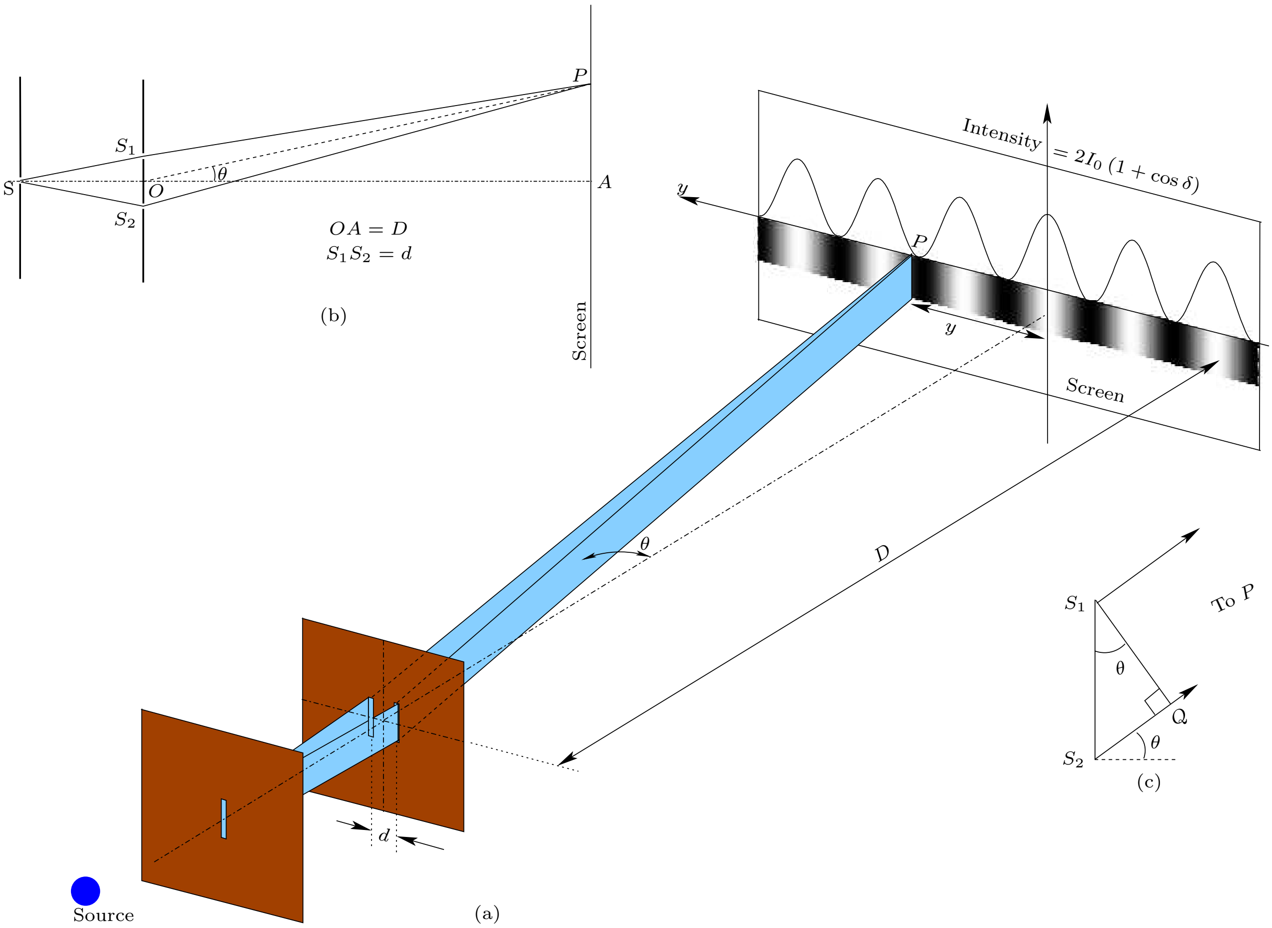

Figure 8: (a) The experimental set up of Young's double slit experiment showing a monochromatic source. An opaque screen with a thin slit lets a wavefront to spread to the next slit with two slits and the result is observed on the screen. The figure also shows the pattern of alternate bright and dark *fringes* observed on the screen and how the intensity varies as function of y. (b) and (c) show the calculation of the path difference.

are illuminated by another narrow slit S which is placed symmetrical with respect to S_1 and S_2 and is, in turn, illuminated by a bright source. Light waves spread out from S and fall on both S_1 and S_2. S_1 and S_2 then behave like two coherent sources because they have been derived from the same source and having traveled the same distance maintain the same phase. Light now spreads out from both S_1 and S_2 and falls on a screen. It is essential that the waves from the two sources overlap on the same part of the screen. If one slit is covered up, the other produces a wide smoothly lit patch on the screen. But when both are open, the screen is covered with alternate dark and bright bands which are called *interference fringes*. A part of the screen which received light from either S_1 and S_2 alone can become dark when both slits are open!

In order to understand this phenomena quantitatively, we turn to the analysis of the process.

3.2.1 YDSE – Quantitatively

Let us say that light is traveling in a medium with refractive index n. In this medium, its speed is $v = c/n$. Suppose it goes from the point A to B, the distance between the two points being s.

The time taken by light to travel this distance $t = s/v$. Since in one time period T, the field vector complete one cycle of oscillation. Therefore, in time t, the number of oscillations that the field vector will undergo is t/T. Further, the phase associated with one oscillation is 2π. Thus the *gain in phase*, $\Delta\varphi$ associated with the number t/T of oscillations is given by

$$\Delta\varphi = 2\pi\,\frac{t}{T} = 2\pi\,\frac{s}{vT} = 2\pi\,\frac{s}{\frac{c}{n}\,\frac{1}{\nu}} = \frac{2\pi}{\lambda}\,ns,$$

where I have used the fact that $c = \lambda\nu$. The above expression can be written in two forms:

$$\Delta\varphi = \frac{2\pi}{\lambda} \times n \times (\text{physical path length covered by light}), \tag{28}$$

$$\text{or} \quad \Delta\varphi = \frac{2\pi}{\lambda} \times (\text{optical path length covered by light}). \tag{29}$$

Notice that in both the expressions, the wavelength is that measured in vacuum, the effect of the properties of the medium being taken into account by the number n. By comparing the two expressions, we get that

$$\text{optical path length} = (\text{refractive index}) \times (\text{physical path length}).$$

Finally, if the light wave starts at A with an initial phase of φ_0, then its phase when it reaches B (along the straight line AB) is

$$\varphi = \varphi_0 + \Delta\varphi = \varphi_0 + \frac{2\pi}{\lambda}\,n \cdot AB.$$

We ca now apply this concept of path length for the YDSE set up. Let us first consider the case as shown in Figure 8(b), when S is placed symmetrically with respect to S_1 and S_2. In this case, there is no phase difference between the two light rays reaching S_1 and S_2, because their path lengths (physical or optical) are same. Let us concentrate at a typical point P on the screen that receives light from both S_1 and S_2. We set up our coordinate system so that the point A lying on the straight line joining S and the center O of S_1S_2 is the origin and AP is the positive y axis. Further let the distance of P from A be y. The (average) intensity I on the screen depends on the coordinate y as per the relation

$$I(y) = 2I_0\,(1 + \cos\delta), \tag{30}$$

where δ is the phase difference between the two waves reaching the point P and it depends on y.

Also the phase difference δ will be given by the relation

$$\delta = \frac{2\pi}{\lambda} \times (S_2P - S_1P),$$

since $n = 1$ here. As shown in Figure 8(c), S_2P and S_1P are nearly parallel since the distance $S_1S_2 = d$ is much less than $OA = D$. The angle that these two lines make with the normal to the screen is denoted by θ. A perpendicular S_1Q is dropped from S_1 onto S_2P. The angle subtended at P by S_1Q is very small. We can therefore think of S_1Q as the arc of a circle centered at P having a radius PS_1. The length PQ then is equal to PS_1. And hence the desired path difference

$$\begin{aligned} S_2P - S_1P &= S_2P - QP \\ &= S_2Q = d\sin\theta \\ &\approx d\tan\theta = d\frac{y}{D}. \end{aligned} \tag{31}$$

In the last expression, use has been made of the fact that for $\theta \ll 1$, $\sin\theta \approx \theta \approx \tan\theta$. Thus, from (30), we get the phase difference

$$\delta = \frac{2\pi}{\lambda}\frac{d}{D}\,y, \tag{32}$$

and hence the intensity on the screen

$$I(y) = 2I_0\left[1+\cos\left(\frac{2\pi}{\lambda}\frac{d}{D}\,y\right)\right], \tag{33}$$

where I_0 is the intensity produced by an individual slit open.

This intensity will be a *maximum* at all points where the cosine factor is equal to its maximum value which equals 1. This happens whenever y is such that the factor $\frac{2\pi}{\lambda}\frac{d}{D}\,y$ is *an even multiple of* π. Thus we obtain,

$$\text{maximum intensity at} \quad y_m = m\lambda\frac{D}{d}, \qquad m \in \mathbf{I}, \tag{34}$$

where $\mathbf{I}$ denotes the set of all integers. The subscript for y implies the position corresponding to different integers. Obviously, $m = 0$ implies the position at which the two waves interfere with *zero phase difference*. This position is called the *central maximum*. For the present situation, the central maximum coincides with A.

This intensity will be a *minimum* at all points where the cosine factor is equal to its minimum value which equals -1. This happens whenever y is such that the factor $\frac{2\pi}{\lambda}\frac{d}{D}\,y$ is *an odd multiple of* π. Thus we obtain,

$$\text{minimum intensity at} \quad y_m = (2m+1)\lambda\frac{D}{d}, \qquad m \in \mathbf{I}. \tag{35}$$

The *fringe width* is the separation between two successive maxima (or minima) and is found by subtracting the values of y corresponding to successive values of m. Thus the fringe width, Δy is given by

$$\Delta y = (m+1-m)\lambda\frac{D}{d}. \tag{36}$$

This separation subtends an angle $\Delta\theta$ at the center O of the slits which is given by:

$$\Delta\theta = \frac{\Delta y}{D} = \frac{\lambda}{d}. \tag{37}$$

The angular separation of the fringes is thus independent of the position of the screen. This is the increase in θ needed to increase the path difference by λ.

So far we have placed the first slit S symmetrically with respect to the two slits S_1 and S_2. This produced zero path difference at the point A on the screen. We now place it at a point specified by the ordinate y_1 with respect to O_1 as shown in Figure 9. We assume additionally that the distance $OO_1 = D_1$ is far greater than $S_1S_2 = d$. Then the line SS_1 and SS_2 are almost parallel and make an angle α with the horizontal. On the screen, at the location of the central maximum, the total path difference must be zero. Thus we have

$$SS_2 + S_2P - SS_1 - S_1P = (S_2P - S_1P) + (S_2S - S_1S) = 0;$$

which means that

$$d\sin\alpha + d\sin\theta = 0 \quad \Rightarrow d\tan\alpha + d\tan\theta = 0 \quad \Rightarrow y_1\frac{d}{D_1} + y\frac{d}{D} = 0.$$

Here once again the small angle approximation has been used to replace the sines by the corresponding tangents. From the last expression, we obtain

$$\frac{y_1}{D_1} = -\frac{y}{D} \qquad \text{or} \quad \theta = -\alpha. \tag{38}$$

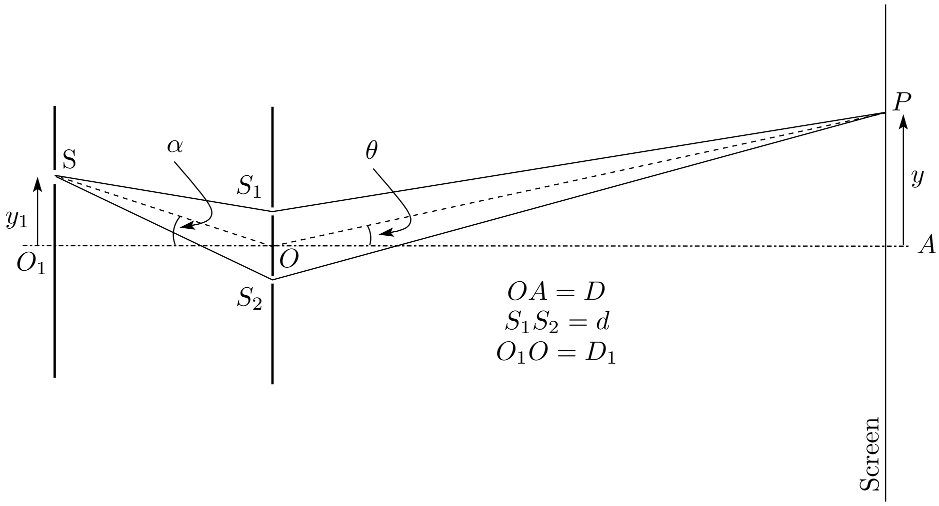

Figure 9: Young's experiment with a different set up.

The meaning of Equation 38 is that *the angular movement θ of the central bright fringe is equal and opposite to the angular movement α of the source.* Both the angles are measured at the center of the double slits. More simply stated, *the first slit, the center of the double slits and the central fringe lie in a straight line as the source is moved.*