Assignment
(Oscillations)

1. Which of the following examples represent (nearly) simple harmonic motion, and which represent periodic but not simple harmonic motion:

1. the rotation of Earth about its axis,
2. motion of an oscillating mercury column in a U–tube,
3. motion of ball inside a smooth curved bowl, when released from a point slightly above the lowermost position,
4. *general* vibration of a polyatomic molecule about its equilibrium configuration.

2. Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non–periodic motion? Give the period for each case of periodic motion: (ω is a positive constant)

(i) $\sin\omega t - \cos\omega t$

(ii) $\sin^3\omega t$

(iii) $3\cos(\pi/4 - 2\omega t)$

(iv) $\cos\omega t + \cos 3\omega t + \cos 5\omega t$

(v) $e^{-\omega^2 t^2}$

(vi) $1 + \omega t + \omega^2 t^2$

3. A particle is in linear simple harmonic motion between two points A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the *signs* of velocity, acceleration, and force on the particle when it is

(i) at the end A,

(ii) at the end B,

(iii) at the mid–point of AB going towards A,

(iv) 2 cm away from B going towards A,

(v) 3 cm away from A going towards B, and

(vi) 4 cm away from A going towards A.

4. The displacement of a particle as a function of time t is given by the expression

$$x(t) = (4.00\,\text{m})\cos(3.00\pi t + \pi),$$

where x is in meters and t is in seconds. Determine (a) the frequency and period of the motion, (b) the amplitude of the motion, (c) the phase constant, and (d) the displacement of the particle at $t = 0.250$ s.

5. A ball dropped from a height of 4.00 m makes a perfectly elastic collision with the ground. Assuming that no energy is lost due to air resistance, (a) show that the motion is periodic and (b) determine the period of the motion. (c) Is the motion simple harmonic? Explain.

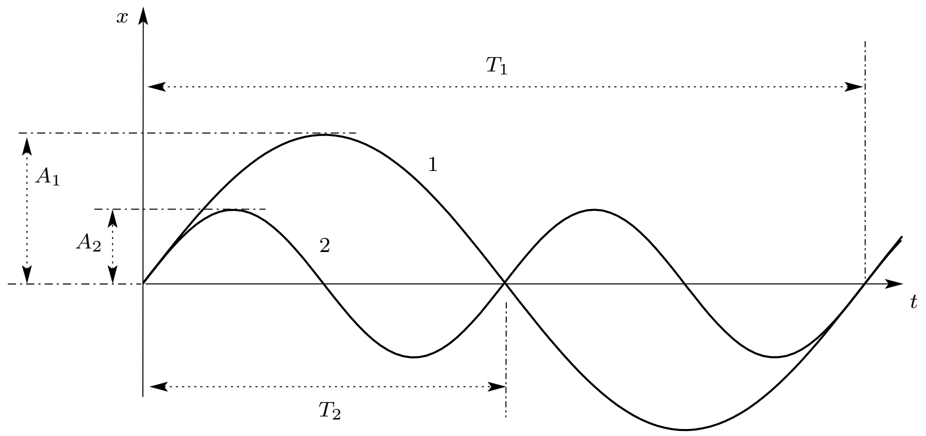
6. A particle moves in simple harmonic motion with a frequency of 3.00 oscillations/s and an amplitude of 5.00 cm. (a) Through what total distance does the particle move during one cycle of its motion? (b) What is its maximum speed? Where does this occur? (c) Find the maximum acceleration of the particle. Where in the motion does the maximum acceleration occur?

7. A 0.500 kg mass attached to a spring with a force constant of 8.00 N/m vibrates in simple harmonic motion with an amplitude of 10.0 cm. Take the origin at the equilibrium position. Calculate (a) the maximum value of its speed and acceleration, (b) the speed and acceleration when the mass is 6.00 cm from the equilibrium position, and (c) the time it takes the mass to move from $x = 0$ to $x = 8.00$ cm.

8. A particle that hangs from a spring oscillates with an angular frequency ω. The spring-particle system is suspended from the ceiling of an elevator car and hangs motionless (relative to the elevator car) as the car descends at a constant speed v. The car then stops suddenly. (a) With what amplitude does the particle oscillates? (b) What is the equation of motion for the particle? (Choose upward as the positive direction.)

9. The amplitude of a system moving in simple harmonic motion is doubled. Determine the change in (a) the total energy, (b) the maximum speed, (c) the maximum acceleration, and (d) the period.

10. Two particles of equal mass are performing simple harmonic oscillations whose position (from the equilibrium) versus time graphs are shown in the figure. Which one has a higher total energy?



11. A "seconds" pendulum is one that moves through its equilibrium position once each second. (The period of the pendulum is 2.000 s.) The length of a seconds pendulum is 0.9927 m at Tokyo and 0.9942 m at Cambridge, England. What is the ratio of the free-fall accelerations at these two locations?

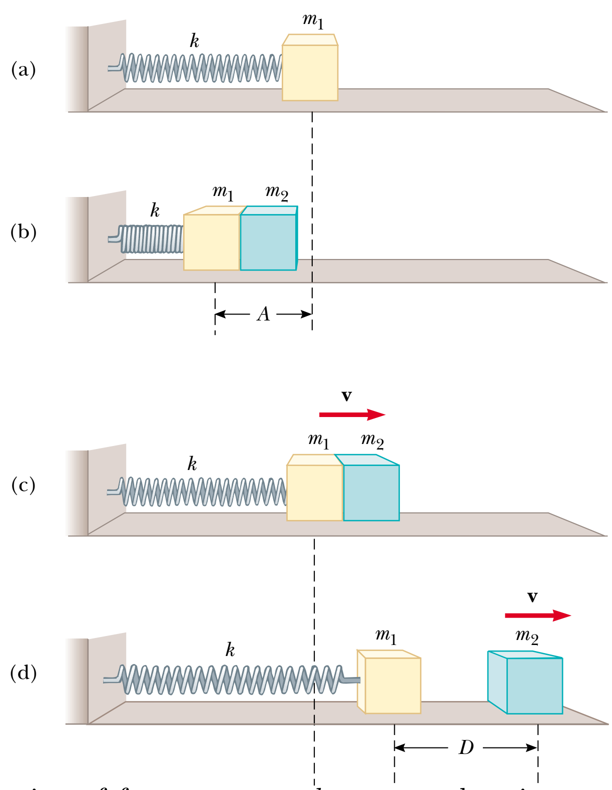
12. A block performs small oscillations in a vertical plane while moving without friction over the internal surface of a spherical cup. Determine the period of oscillations of the block if the internal radius of the cup is R and the face of the block is much smaller than R.

13. How will the period of oscillations of the block in the last problem change, if, besides the force of gravity, the cup is acted upon by a force F directed vertically upward? The mass of the cup M is much greater than that of the block m.

14. A physical pendulum in the form of a planar body moves in simple harmonic motion with a frequency of 0.450 Hz. If the pendulum has a mass of 2.20 kg and the pivot is located 0.350 m from the center of mass, determine the moment of inertia of the pendulum.

15. A very light, rigid rod with a length of 0.500 m extends straight out from one end of a meter stick. The stick is suspended from a pivot at the far end of this light rod and is set into oscillation. (a) Determine the period of oscillation. (b) By what percentage does this differ from a 1.00 m long simple pendulum?

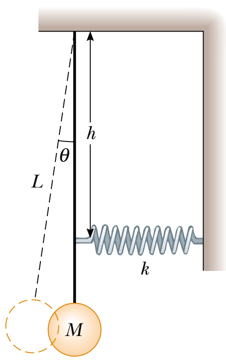
16. A mass, $m_1 = 9.00$ kg, is in equilibrium while connected to a light spring of constant $k = 100$ N/m that is fastened to a wall, as shown in part (a) of the adjacent diagram. A second mass, $m_2 = 7.00$ kg, is slowly pushed up against mass m_1, compressing the spring by an amount $A = 0.200$ m (see part (b)). The system is then released, and both masses start moving to the right on the frictionless surface. (i) When m_1 reaches the equilibrium point, m_2 loses contact with m_1 (see part (c)) and moves to the right with speed v. Determine the value of v. (ii) How far apart are the masses when the spring is fully stretched for the first time (D in part (d))?

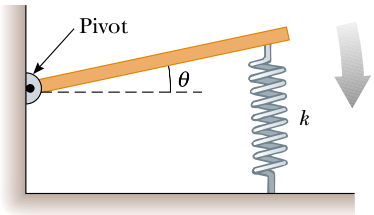


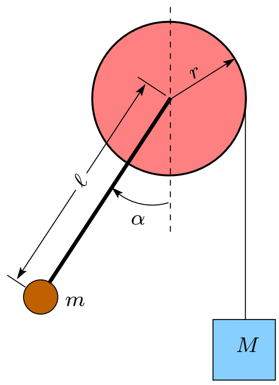
17. A spherical ball of mass m and radius r rolls without slipping inside a fixed hemispherical bowl of radius R. Find the period of small oscillations of the smaller sphere about its equilibrium position.

18. A pendulum of length L and mass M has a spring of force constant k connected to it at a distance h below its point of suspension (see figure). Find the frequency of vibration of the system for small values of the amplitude (small θ). (Assume that the vertical suspension of length L is rigid, but neglect its mass.)

19. A horizontal uniform rod of mass m and length L is pivoted at one end. The rod's other end is supported by a spring of force constant k (see figure). Show that the rod, after being displaced a small angle θ from its horizontal equilibrium position and released, executes simple harmonic motion and find the time period.







20. A massless rod of length ℓ is rigidly attached to a massless pulley of radius r. The end of the rod carries a mass m. A string whose free end carries a mass M is wounded over the pulley. Find the condition for which the system will undergo oscillatory motion if initially the angle α is zero.