Assignment on Electrostatics

Mutltiple Choice Questions

1. Each of the following figures shows electric field vectors at two points in a region of an electric field. In which figure or figures can the illustrated field be created by a single point charge?

2. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical just after release. Then X is nearly

(A) 10^{-5} V (B) 10^{-7} V (C) 10^{-9} V (D) 10^{-10} V

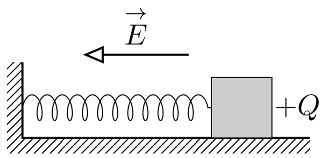
3. A wooden block performs SHM on a frictionless surface with frequency ν_o. The block contains a charge $+Q$ on its surface. If now a uniform electric field $\vec{E}$ is switched on as shown, then SHM of the block will be

(A) of the same frequency and with shifted mean position.

(B) of the same frequency and with same mean position.

(C) of changed frequency and with shifted mean position.

(D) of changed frequency with same mean position.



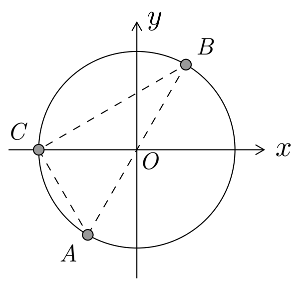
4. Consider a system of three charges $\frac{q}{3}$, $\frac{q}{3}$ and $-\frac{2q}{3}$ placed at points A, B and C, respectively, as shown in the adjoining figure. Take O to be the center of the circle of radius R and $\angle CAB = 60°$.

(A) The electric field at point O is $\frac{q}{8\pi\epsilon_o R^2}$ directed along the negative x-axis.

(B) The potential energy of the system is zero.

(C) The magnitude of the force between the charge C and B is $\frac{q^2}{54\pi\epsilon_o R^2}$.

(D) The potential at O is $\frac{q}{12\pi\epsilon_o R}$.



5. When identical point charges are placed at the vertices of a cube of edge length a, each of them experiences a net force of magnitude F. Now these charges are placed on the vertices of another cube of edge length b. What will be the magnitude of the net force on any one of the charges?

(A) $\frac{a^2F}{b^2}$ (B) $\frac{b^2F}{a^2}$ (C) $\frac{aF}{b}$ (D) Insufficient information

6. The potential due to a point charge at a point A is 7 V and the electric field there is 3 V/m. There is another point B such that the electric field at this point has smaller magnitude as compared to that at point A. However, if the magnitude of the charge is tripled, the electric field at B becomes 3 V/m. The potential at B now is closest to

(A) 7/3 V (B) 7 V (C) 12 V (D) 21 V

7. Positive and negative point charges of equal magnitude are kept at $(0, 0, a/2)$ and $(0, 0, -a/2)$, respectively. The work done by the electric field when another positive point charge is moved from $(-a, 0, 0)$ to $(0, a, 0)$

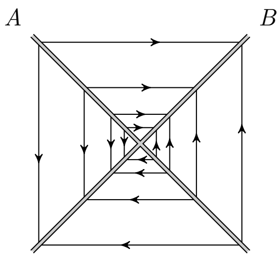
(A) is positive.

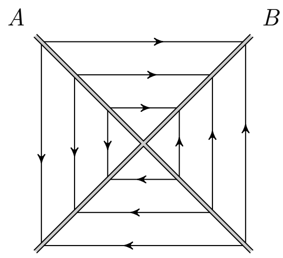
(B) is negative.

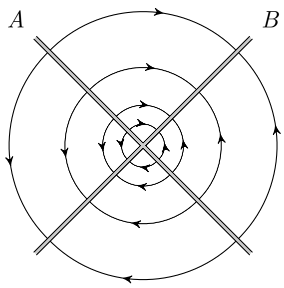
(C) is zero.

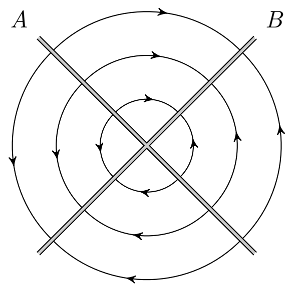
(D) can be positive or negative depending on the path taken by the charge.

8. Two infinitely large planes A and B intersect each other a right angles and carry uniform surface charge densities $+\sigma$ and $-\sigma$, respectively. In which of the following figures, electric field lines are correctly represented?

(A) 

(B) 

(C) 

(D) 

9. Two charged spheres are kept at finite center-to-center distance. The force of electrostatic interaction between them is first calculated assuming them to be point like charges at their respective centers and then the force is measured experimentally. If the calculated and the measured values are F_c and F_m respectively, on which of the following conclusions can you certainly reach?

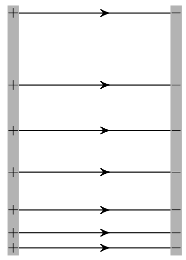
(A) If $F_c > F_m$ with charges of the same sign and $F_c < F_m$ with charges of opposite signs, both the spheres must be made of insulating materials.

(B) If $F_c > F_m$ with charges of same sign and $F_c < F_m$ with charges of opposite signs, both the spheres must be made of conducting materials.

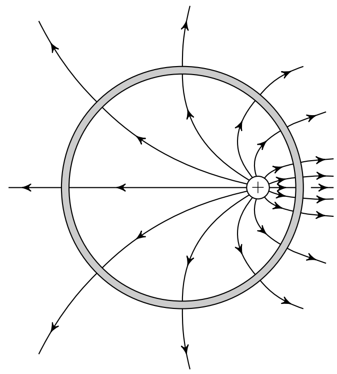
(C) Irrespective of their materials, $F_c < F_m$ when they carry charges of the same sign, and $F_c > F_m$ when they carry charges of opposite signs.

(D) Irrespective of their materials, $F_c > F_m$ when they carry charges of the same sign, and $F_c < F_m$ when they carry charges of opposite signs.

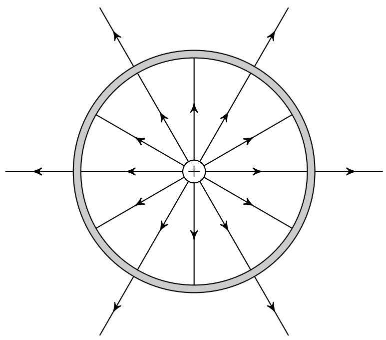
10. Which of the following field lines are incorrect representations?



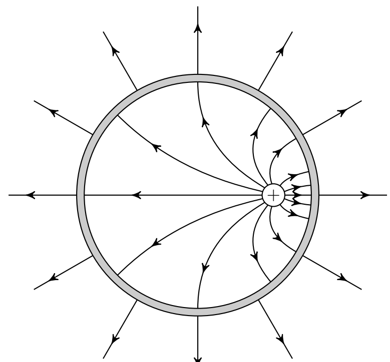
(A) Two infinitely large non conducting sheets



(B) Point charge inside a neutral conducting shell



(C) Point charge inside a neutral conducting shell



(D) Point charge inside a neutral conducting shell

11. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to

(A) x (B) x^2 (C) x^3 (D) $1/x$

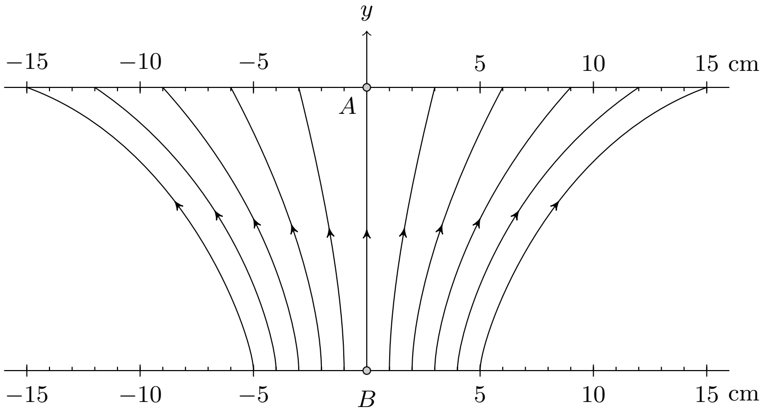
12. A charge q is placed at the center of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to

(A) $-Q/2$ (B) $-Q/4$ (C) $+Q/4$ (D) $+Q/2$

13. Two identical thin rings, each of radius R, are co-axially placed a distance R apart. If Q_1 and Q_2 are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the center of one ring to that of the other is

(A) 0 (B) $\dfrac{(\sqrt{2}-1)q(Q_1-Q_2)}{\sqrt{2}(4\pi\epsilon_o R)}$ (C) $\dfrac{\sqrt{2}q(Q_1+Q_2)}{4\pi\epsilon_o R}$ (D) $\dfrac{(\sqrt{2}+1)q(Q_1+Q_2)}{\sqrt{2}(4\pi\epsilon_o R)}$

14. A charged particle is in equilibrium at point A under combined action of a uniform gravitational field and a non-uniform electric field. The electric field is symmetric with respect to rotation about the y axis and the lines of the electric field in a plane are shown in the figure below:



The magnitude of the charge on the particle is gradually changed till it shifts to another equilibrium position B. The fractional change in the charge of the particle is

(A) $\dfrac{1}{3}$ (B) $\dfrac{2}{3}$ (C) $\dfrac{1}{9}$ (D) $\dfrac{8}{9}$

15. Consider a cube as shown in Figure I, with uniformly distributed charge within its volume. At one of the corners P, the electric field magnitude is E_o and the potential is V_o. A portion of half the size (half side length) of the original cube is removed as shown in Figure II.

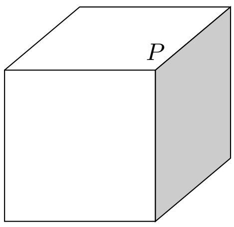


Figure I

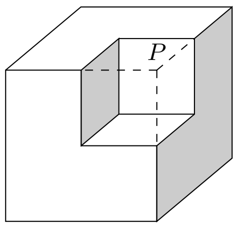


Figure II

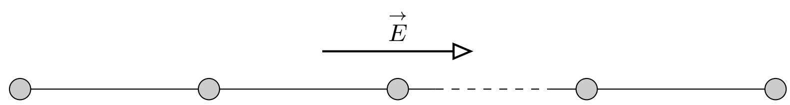
What would now be the magnitude of the electric field and potential at the point P?

(A) $\dfrac{7E_o}{8}, \dfrac{7V_o}{8}$ (B) $\dfrac{3E_o}{4}, \dfrac{V_o}{2}$ (C) $\dfrac{E_o}{2}, \dfrac{3V_o}{4}$

16. The electrostatic potential V at a point on the circumference of a circular layer carrying a uniform surface charge density σ and radius R is given by $V = \dfrac{\sigma R}{\pi\epsilon_o}$. What would be the electrostatic potential energy of such a circular layer?

(A) $\dfrac{\sigma^2 R^3}{\epsilon_o}$ (B) $\dfrac{8\sigma^2 R^3}{3\epsilon_o}$ (C) $\dfrac{2\sigma^2 R^3}{5\epsilon_o}$ (D) $\dfrac{2\sigma^2 R^3}{3\epsilon_o}$

17. A straight chain consists of n identical metal balls is at rest in a region of free space as shown in the figure. In the chain, each ball is connected to next ball with identical thin conducting wire. Length ℓ of each connecting wire is much larger than the radius r of each of the balls.



Now a uniform electric field of magnitude E and parallel to the chain is switched on in the region. The magnitude of the charge appearing on the left-most ball is

(A) $2\pi\epsilon_o rn\ell E$

(B) $2\pi\epsilon_o r(n-1)\ell E$

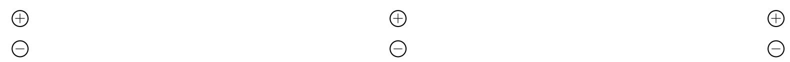
(C) $4\pi\epsilon_o r(n-1)\ell E$

(D) $4\pi\epsilon_o r(n-1)^2\ell^2 E$

18. If a point dipole $p\,\hat{i}$ and a point charge placed at the points $(0, y)$ and at the origin, respectively, create no net electric field at the point $(x, x+y)$ (where $x, y > 0$), then y must be equal to

(A) x (B) $2x$ (C) $2.5x$ (D) $3x$

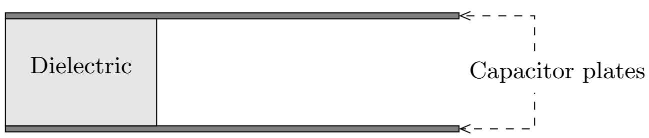
19. Three identical small dipoles are arranged at equal distances as shown in the figure below:



The separation between the charges of the dipoles is negligible as compared to separation between dipoles. In the given configuration, their total electrostatic interaction energy is U_o. Now one of the end dipoles is gradually reversed. For this process, the work done by the electric forces is

(A) $\dfrac{17}{18}U_o$ (B) $-\dfrac{17}{18}U_o$ (C) $\dfrac{18}{17}U_o$ (D) $-\dfrac{18}{17}U_o$

20. A parallel plate capacitor of capacitance C_o is fully charged by a battery of emf $\mathcal{E}$ and then disconnected. A dielectric covering one-third area of the each plate is inserted as shown below:



The charge on the capacitor plates gets redistributed such that the region not covered with dielectric and the region covered with dielectric share equal amount of charge. Then choose the correct statement/s.

(A) The dielectric constant of the dielectric is 2.0

(B) Charge appearing due to polarization on the surface of the dielectric is $\frac{1}{4}C_o\mathcal{E}$

(C) Force of electrostatic interaction between portions of the plates covered with dielectric equals to that between uncovered portions.

(D) The force of electrostatic interaction between the plates after insertion of the dielectric becomes $\frac{8}{9}$ times of its value before the insertion of the dielectric.

Descriptive Questions

1. A stationary charge distribution produces a radial electric field given by

$$\vec{E} = A\frac{e^{-br}}{r}\,\hat{r}$$

where A and b are positive constants and r is the distance from the origin. Determine (i) the total charge of the system; and (ii) the volume density of the charge as a function of r.

2. Suppose that instead of the Coulomb's law, it is found that the electrostatic force (in magnitude) between two point charges q_1 and q_2, which are separated by a distance r, is given by the formula

$$F = \frac{q_1 q_2}{4\pi\epsilon_0}\frac{(1-\sqrt{\alpha r})}{r^2}$$

where α is a positive constant. However, like the Coulomb force, this force is also central and is attractive for unlike charges and repulsive for like charges.

(i) Write down the appropriate electric field $\vec{E}$ surrounding a point charge q.

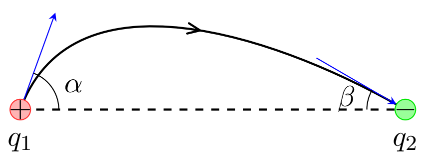
(ii) Take a closed path $\mathcal{C}$ around this point charge and evaluate $\oint_{\mathcal{C}} \vec{E}\cdot \mathrm{d}\vec{\ell}$. Compare with the Coulomb result.

(iii) Find $\oint_{\mathcal{S}} \vec{E}\cdot \mathrm{d}\vec{A}$ over a spherical surface $\mathcal{S}$ of radius R centred at the point charge q. Compare with the Coulomb result.

3. Two small identical balls lying on a smooth horizontal plane are connected by a mass-less spring. One ball is fixed at point O and the other is free. The balls are charged identically, as a result of which the spring length increases twofold. Determine the change in the frequency of small harmonic vibrations of the system.

4. The inside of a grounded spherical metal shell (inner radius R_1 and outer radius R_2) is filled with space charge of uniform volume density ρ of charge. Find the potential at the center and the electrostatic energy of the system.

5. An electric field line emerges from a positive point charge $+q_1$ making an angle α with the straight line joining it to a negative point charge $-q_2$ as shown below. At what angle β will this field line enter the charge $-q_2$?



6. Determine the magnitude E of the electric field at the center of a hemisphere produced by charges uniformly distributed with a density σ over the surface of this hemisphere.

7. Determine the force F of interaction between two hemispheres of radius R touching each other along the equator if one hemisphere is uniformly charged with a surface density σ_1 and the other with a surface density σ_2.

8. Two small balls of mass m, bearing a charge q each, are connected by a non-conducting thread of length 2ℓ. Starting at a certain instant, the mid-point of the thread starts moving at a constant velocity v perpendicular to the direction of the initial orientation of the thread. Determine the minimum distance between the balls.

9. Three identical point charges are placed at the vertices of an equilateral triangle. At how many places will the net electric field zero within the triangle?

10. Two particles of charges and masses $(+q_1, m_1)$ and $(-q_2, m_2)$ are released at two different points in a uniform electric field E established in free space. If their separation remains unchanged, determine it.

11. Four point charges $+8\mu$C, -1μC, -1μC, and $+8\mu$C are fixed at the points $-\sqrt{27/2}$ m, $-\sqrt{3/2}$ m, $+\sqrt{3/2}$ m and $+\sqrt{27/2}$ m respectively on the y-axis. A particle of mass 6×10^{-4} kg and charge $+0.1\mu$C moves along the x-direction. Its speed at $x = +\infty$ is v_o. Find the least value of v_o for which the particle will cross the origin. Assume that space is gravity free.

12. Three identical thin uniformly charged filaments are fixed along the sides of a cube as shown in the Figure 12. Length of each filament is ℓ and each of them carry a linear charge density λ. Determine the electric field at the center of the cube. The cube is just a geometrical construct and not made of any matter.

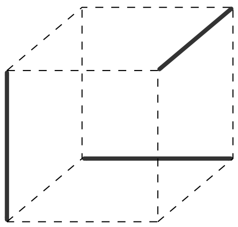


Figure 1: Problem 12

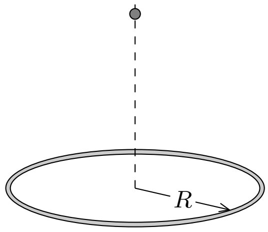


Figure 2: Problem 13

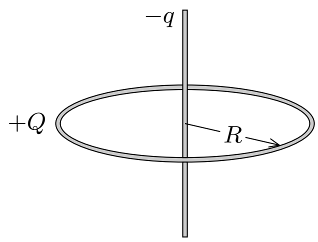


Figure 3: Problem 14

13. A uniformly charged ring of radius R is fixed near the Earth's surface with its plane horizontal (Figure 2). The total charge on the ring is Q. Consider a small charged bead of mass m on the vertical axis of the ring. If it is displaced slightly from a particular position, it can undergo oscillatory motion. Determine the charge on the bead.

14. In free space, a thin rod carrying uniformly distributed negative charge $-q$ is placed symmetrically along the axis of a thin ring of radius R carrying a uniformly distributed charge Q (see Figure 14). Mass of the rod is m and its length $\ell = 2R$. The ring is fixed but the rod is free to move. The rod is displaced slightly along the axis and then released. Find the period T of small oscillations of the rod.

15. An isolated soap bubble of radius 1 cm is at a potential of 100 volts. If it collapses to a drop of radius 1 mm, what is the change of its electrostatic energy?

16. Small identical balls with equal charges are fixed at the vertices of a regular 2018-gon with side a. At a certain instant, one of the balls is released, and a sufficiently long time interval later, the ball adjacent to the first released ball is freed. The kinetic energies of the released balls are found to differ by K at a sufficiently long distance from the polygon. Determine the charge q of each ball.

17. A very small earthed conducting sphere is at a distance a from a point charge q_1 and at a distance b from a point charge q_2 $(a < b)$. At a certain instant, the sphere starts expanding so that its radius grows according to the law $R = vt$. Determine the time dependence $I(t)$ of the current in the earthing conductor, assuming that the point charges and the center of the sphere remain at rest, and in due time the initial point charges get into the expanding sphere without touching it (through small holes).

18. Three small identical neutral metal balls are at the vertices of an equilateral triangle. The balls are in turn connected to an isolated large conducting sphere whose center is on the perpendicular erected from the plane of the triangle and passing through its center. As a result, the first and second balls acquired charges q_1 and q_2 respectively. Determine the charge of the third ball.

19. A conducting sphere of radius $R = 1.0$ m is charged to a potential $V_i = 1000$ V. A thin metal disk of radius $r = 1.0$ cm mounted on an insulated handle is touched with the sphere making contact with one of its flat faces and then separated. After separation the disk is earthed and the process is repeated until the potential of the sphere becomes $V_f = 999$ V. Approximately how many times has the process been repeated?

20. Two metal spheres of radii r_1 and r_2 are connected by a thin conducting wire. The second sphere is surrounded by a concentric conducting shell with uniform separation d between their facing surfaces, and connected to the ground (Figure 4). The second sphere is given a charge Q. Find charges acquired by all three spheres. Assume that $d \ll r_2$ and the distance between the spheres is much larger than their radii.

Figure 4: Problem 20

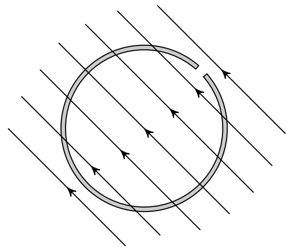


Figure 5: Problem 23

21. A thin rigid insulating ring of radius r and mass m has a very small gap of length ℓ (where $\ell \ll r$) and carries a charge q distributed uniformly on it. Initially the ring is at rest in a gravity free space when an electric field of magnitude E parallel to the plane of the ring and directed parallel to the gap is switched on as shown in Figure 5. Determine the maximum angular speed acquired by the ring during subsequent motion.

22. In the Figure 6, determine the potential V_0 at the point O if the potentials of the points A, B, and D are, respectively V_1, V_2, and V_3. The capacitors were initially uncharged.

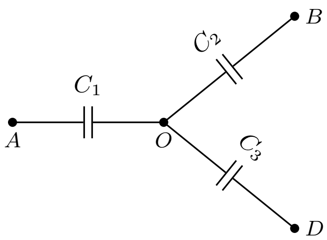


Figure 6: Problem 22

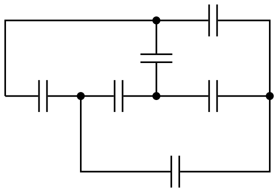


Figure 7: Problem 23

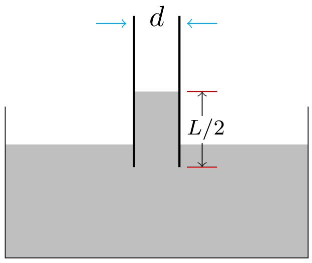


Figure 8: Problem 24

23. Across which two points should an emf be connected in the circuit shown in Figure 7 in order to charge all the six capacitors having equal capacitance?

24. A parallel plate capacitor with square plates of side L and plate separation d is charged to a potential V_0 and disconnected from the battery. It is then vertically inserted into a large reservoir of dielectric liquid with dielectric constant k and density ρ until the liquid fills half the space between the capacitor plates as shown in Figure 8.

(i) Find the capacitance of the capacitor when the liquid has risen in between its plates.

(ii) What is the electric field strength between the capacitor plates?

(iii) What is the distribution of charge density over the plates?

(iv) What is the difference in height between the level of liquid within the capacitor plates and that in the external reservoir?

25. Figure 9 shows an "air gap" capacitor for manual tuning. Alternate plates are connected together; one group of plates is fixed in position, and the other group is capable of rotation. (i) Consider 8 such plates each having area of 1.25 cm^2 and separated from adjacent plates by a distance d = 3.40 mm. What is the maximum capacitance of the device? (ii) Obtain an expression for the maximum capacitance for the case of $2n$ plates, each with area A and separation between adjacent plates being d.

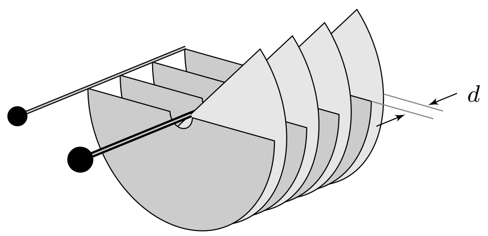


Figure 9: Problem 25

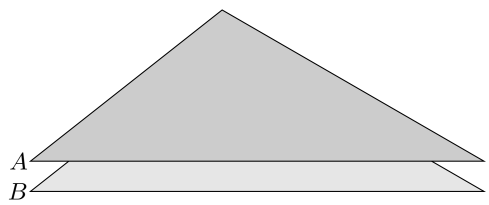


Figure 10: Problem 26

26. Two equilateral triangular plates are placed parallel to each other. The separation between them is much smaller than their linear dimensions and they overlap completely as shown in Figure 10. One of them carries a uniform positive charge and the other an equal amount of uniform negative charge. If the electric field magnitude well inside the plates is E_o, find the electric field magnitude at the mid-point of the line AB. What happens if the plates have the shape of a regular pentagon?

27. A grounded metallic sphere of radius r is placed touching the center of a uniformly charged thin insulating disk of radius R (see Figure 11). The disk is so large that the radius r of the sphere can be neglected as compared to the radius R of the disk. Total charge on the disk is Q. Find the force of electrostatic interaction between the sphere and the disk.

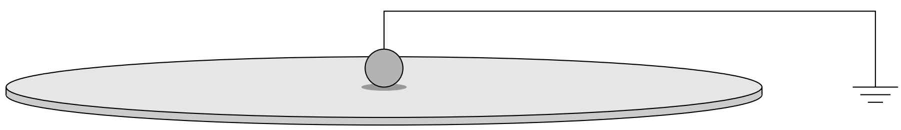


Figure 11: Problem 27

28. A parallel plate capacitor is charged to a voltage $V_0 = 156$ V and connected in parallel to another uncharged parallel plate capacitor which has the same dimensions as the first one but is filled with a dielectric whose dielectric constant depends on the voltage applied across the capacitor as $k = \alpha V$, where $\alpha = 1$ per volt. Determine the final voltage of the capacitors.

29. The volume between two concentric conducting spherical surfaces of radii a and b (where $a < b$) is filled with an inhomogeneous dielectric with dielectric constant k depending on the distance r from the center of the spheres as $k = \dfrac{k_o}{1 + \alpha r}$. Determine the capacitance of the system.

30. Charge given to a conducting sphere distributed uniformly on its surface. How will a charge given to a very thin conducting disk (2-dimensional) and a thin conducting needle (1-dimensional) distribute? Justify.

Answers

Multiple Choice Questions

1.	(A), (C)	**2.**	(C)	**3.**	(A)	**4.**	(C)	**5.**	(A)
6.	(C)	**7.**	(C)	**8.**	(B)	**9.**	(D)	**10.**	(A), (B), (C)
11.	(B)	**12.**	(B)	**13.**	(B)	**14.**	(D)	**15.**	(C)
16.	(D)	**17.**	(B)	**18.**	(B)	**19.**	(C)	**20.**	(A), (B), (D)

Descriptive Questions

1. (i) 0, (ii) The system consists of a point charge $4\pi\epsilon_0 A$ at the origin and rest of the space filled with a volume density $-\dfrac{\epsilon_0 Ab}{r^2}e^{-br}$

2. (i) $E = \dfrac{q}{4\pi\epsilon_0}\dfrac{1}{r^2}(1-\sqrt{\alpha r})$ pointing radially. (ii) The line integral is 0 same as Coulomb's law. (iii) The flux is $\dfrac{q}{\epsilon_0}(1-\sqrt{\alpha R})$.

3. New frequency $= \sqrt{2}\times$ old frequency

4. $V_c = \dfrac{\rho R_1^2}{6\epsilon_0}$, electrostatic energy $= \dfrac{2\rho^2 R_1^5}{45\epsilon_0}$

5. $\beta = 2\sin^{-1}\left(\sqrt{\dfrac{q_1}{q_2}}\sin\dfrac{\alpha}{2}\right)$

6. $E = \sigma/4\epsilon_0$

7. $\pi R^2\sigma_1\sigma_2/2\epsilon_0$

8. $2\ell q^2/(q^2+8\pi\epsilon_0 mv^2\ell)$

9. Four

10. $\sqrt{\dfrac{q_1q_2(m_1+m_2)}{4\pi\epsilon_o E(q_1m_2+q_2m_1)}}$

11. 3 m/s

12. Zero

13. $q > \dfrac{6\sqrt{3}\pi\epsilon_o mgR^2}{Q}$

14. $T = 4\pi R\sqrt{\dfrac{2\sqrt{2}\pi\epsilon_o mR}{qQ}}$

15. 5×10^{-8} J

16. $q = \sqrt{4\pi\epsilon_0 Ka}$

17. $I(t) = \begin{cases} v\left(\dfrac{q_1}{a}+\dfrac{q_2}{b}\right), & \text{for } t < \frac{a}{v} \\ v\dfrac{q_2}{b}, & \text{for } \frac{a}{v}\le t < \frac{b}{v} \\ 0 & \text{for } t \ge \frac{b}{v} \end{cases}$

18. $q_3 = q_2^2/q_1$

19. $n = \dfrac{4R^2(V_i - V_f)}{r^2V_i} = 40$

20. $Q_1 = \dfrac{r_1 d}{r_1 d + r_2^2}Q$, $Q_2 = \dfrac{r_2^2}{r_1 d + r_2^2}Q$, charge on the shell $= -Q_2$

21. $\omega_{\max} = \dfrac{q\ell E}{\pi m r^2}$

22. $V_0 = \dfrac{C_1V_1 + C_2V_2 + C_3V_3}{C_1+C_2+C_3}$

23. One capacitor will always remain uncharged, no matter across which two points a battery is applied.

24. (i) $C = \left(\dfrac{k+1}{2}\right)\epsilon_0\dfrac{L^2}{d}$, (ii) $E = \left(\dfrac{2}{k+1}\right)\dfrac{V_0}{d}$, (iii) In the part filled with fluid, the surface charge density is $\left(\dfrac{2k}{k+1}\right)\dfrac{\epsilon_0 V_0}{d}$, and in the other part, the density is $\left(\dfrac{2}{k+1}\right)\dfrac{\epsilon_0 V_0}{d}$

25. (i) 2.28 pF (ii) $C_{\max} = \dfrac{(2n-1)\epsilon_o A}{d}$

26. In case of triangular plates, $E = \dfrac{E_o}{6}$. In case of pentagonal plates, $E = \dfrac{3E_o}{10}$.

27. $F = \dfrac{Q^2 r}{\pi\epsilon_o R^3}$

28. $V_f = \dfrac{\sqrt{1+4\alpha V_0}-1}{2\alpha} = 12$ V

29. $C = \dfrac{4\pi\epsilon_0 ab}{(b-a)+ab\alpha\ln(b/a)}$

30. For a thin conducting disk of radius R, the surface charge density $\sigma_{\text{disk}} = \dfrac{Q}{2\pi R\sqrt{R^2-r^2}}$, where Q is the total charge given to the disk and r is the distance from the center. For a conducting needle, the charge resides practically uniformly on the needle.