Assignment P–11–1

(Units, Measurements and Dimensional Analysis)

1. Find the number of significant figures in the following:

(a) 0.000567

(b) 9.912408

(c) 3.0067

(d) 0.006700

(e) 2.99792458×10^8

2. Multiply the following and show the result after appropriate rounding off: (a) 9.8 m/s^2 with 18.7 s (b) 9.00×10^{-2} with 13.45.

3. Your backyard has brick walls on both ends. You measure a distance of 23.4 m from the inside of one wall to the inside of the other. Each wall is 29.4 cm thick. How far is it from the outside of one wall to the outside of the other?

4. In an article on the SARS epidemic, the May 7, 2003 *New York Times* discusses conflicting estimates of the disease's incubation period (the average time that elapses from infection to the first symptoms). "The study estimated it to be 6.4 days. But other statistical calculations . . . showed that the incubation period could be as long as 14.22 days." What's wrong here?

5. In the equation $y = A\cos(kx - \omega t)$, obtain the dimensional formula for k and ω. Given x is distance with unit m and t is time with unit s.

6. In the Van der Wall's equation

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

find the dimensions of a and b. The SI units of p, V, R, and T are, respectively, $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}$, m^3, $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \cdot \text{K}^{-1}$ and K.

7. An important milestone in the history of the Universe just after Big Bang is Planck's time t_p, the value of which depends on three fundamental constants:

- the speed of light, $c = 3.00 \times 10^8$ m/s
- Newton's Gravitational constant $G = 6.67 \times 10^{-11}\ \text{m}^3\text{s}^{-2}\text{kg}^{-1}$
- Planck's constant $h = 6.634 \times 10^{-34}\ \text{kg}\,\text{m}^2/\text{s}$

Based on dimensional analysis, estimate the order of magnitude of t_p.

8. In an experiment on measurement of the refractive index of glass, the following observations were made: 1.45, 1.56, 1.54, 1.44, 1.54, and 1.53. Calculate the (i) mean value of the refractive index, (ii) mean absolute error, (iii) relative error, and (iv) percentage error. What would be the true value of the refractive index with error limits?

9. Two rods have lengths measured as (1.8 ± 0.2) m and (17.81 ± 0.6) m. Find their sum with error limits.

10. The original length of a wire is (153.7 ± 0.6) cm. It is stretched to (155.3 ± 0.2) cm. Calculate the elongation in the wire with error limits.

11. The radius of a sphere is measured to be (2.1 ± 0.5) cm. Calculate the surface area with error limits.

12. The radius of a sphere is measured with an error of 2 %. What would be the error in the volume of the sphere?

13. A physical quantity x is calculated from $x = \dfrac{ab^2}{\sqrt{c}}$. Calculate maximum % error in x, when % error in measuring a, b, c are 4, 2 and 3 respectively.

14. A physical quantity x is calculated from the relation $x = \dfrac{a^3b^2}{\sqrt{cd}}$. Calculate maximum percentage error in x, if a, b, c, d are measured respectively with an error of 1 %, 3 %, 4 %, and 2 %.

15. Calculate the percentage error in the resistivity $\rho = \pi r^2 \dfrac{R}{\ell}$, where

$$\begin{aligned} r &= \text{radius of the wire } = (0.26 \pm 0.02)\text{ cm}, \\ \ell &= \text{length of the wire } = (156.0 \pm 0.1)\text{ cm}, \\ R &= \text{resistance of the wire } = (64 \pm 2)\text{ ohm}. \end{aligned}$$

16. The frequency ν of oscillation of a gas bubble from an underwater explosion depends upon the static pressure p of water, density ρ of water, and the total energy E of the explosion. Find, using dimensional analysis, the formula for ν.

17. The Reynolds's number N_R determines whether the flow of fluid is *streamline* or *turbulent*. For a fluid flowing through a pipe, it depends on: (i) the density of the liquid, ρ; (ii) the coefficient of viscosity, η; (iii) the speed of flow, v, in the tube; and (iv) the radius r of the tube. Obtain dimensionally an expression for N_R, if it is known that it is directly proportional to r.

18. This exercise is a famous example worked out by the eminent British fluid dynamicist G. I. Taylor[1]. In a nuclear explosion there is an essentially instantaneous release of energy E in a small region of space. This produces a spherical shock wave, with the pressure inside the shock wave thousands of times greater that the initial air pressure, which may be neglected. How does the radius R of this shock wave grow with time t?

[1] Taylor's name is associated with many phenomena in fluid mechanics: the Rayleigh–Taylor instability, Saffman–Taylor fingering, Taylor cells, Taylor columns, etc.