Practice Problems
(Projectile Motion)

1. The maximum height attained by a projectile and its horizontal range are equal to each other if the projectile is projected at an angle θ given by

(A) $\theta = \tan^{-1}(2)$ (B) $\theta = \tan^{-1}(3)$

(C) $\theta = \tan^{-1}(4)$ (D) $\theta = \tan^{-1}(5)$

2. A person aims a gun at target located at a horizontal distance of 100 m. If the gun imparts a horizontal speed of 500 ms^{-1} to the bullet, at what height above the target must he aim his gun in order to hit it? Take $g = 10$ ms^{-2}

(A) 10 cm (B) 20 cm

(C) 50 cm (D) 100 cm

3. When a projectile is at the highest point of its trajectory, the directions of its velocity and acceleration are

(A) parallel to each other

(B) anti- parallel to each other

(C) inclined to each other at an angle of 45°

(D) perpendicular to each other

4. A ball is projected horizontally with a velocity of 5 m/s from the top of a building 19.6 m high. How long will the ball take to hit the ground

(A) $\sqrt{2}$ s (B) 2 s (C) $\sqrt{3}$ s (D) 3 s

5. A bomb is dropped from an airplane when it is at a height h directly above a target. If the airplane is moving horizontally at a speed v, the distance by which the bomb will miss the target is given by

(A) $2v\frac{\sqrt{h}}{g}$ (B) $v\frac{\sqrt{h}}{g}$ (C) $v\frac{\sqrt{2h}}{g}$ (D) $v\frac{\sqrt{h}}{2g}$

6. A missile is fired for maximum range at your own town from a place in the enemy country 100 km away from your town. If the missile is first detected at its halfway point, how much warning time will you have? Take $g = 10$ ms^{-2}

(A) 100 s (B) $\frac{100}{\sqrt{2}}$ s

(C) $100 \times \frac{\sqrt{3}}{2}$ s (D) 200 s

7. In Q.6 above, what was the speed of the missile when it was detected?

(A) $200\sqrt{2}$ m/s (B) $300\sqrt{2}$ m/s

(C) $400\sqrt{2}$ m/s (D) $500\sqrt{2}$ m/s

8. In Q.6 above, what is the maximum height of the missile?

(A) 2.5×10^4 m (B) 5×10^4 m

(C) 7.5×10^4 m (D) 10^5 m

9. In Q.6, the velocity with which the missile strikes the target is

(A) $10^2 ms^{-1}$ (B) $10^3 ms^{-1}$

(C) $10^4 ms^{-1}$ (D) $10^5 ms^{-1}$

10. An enemy plane is flying horizontally at an altitude of $2km$ with a speed of $300ms^{-1}$. An army man with an anti- aircraft gun on the ground sights the enemy plane when it is directly over head and fires a shell with a muzzle speed of $600ms^{-1}$. At what angle with the vertical should the gun be fired so as to hit the plane?

(A) 30° (B) 45° (C) 60° (D) 75°

11. In Q.10, at what minimum altitude should the enemy plane fly to avoid being hit? Take $g = 10ms^{-2}$

(A) $12.5km$ (B) $13.5km$

(C) $14.5km$ (D) $15.5km$

12. A cricketer can throw a ball to a maximum horizontal distance of $100m$. With the same effort, he throws the ball vertically upwards. What is the maximum height attained by the ball?

(A) $50m$ (B) $60m$ (C) $70m$ (D) $80m$

13. From the top of a tower of height $40m$, a ball is projected upwards with a speed of $20m/s$ at angle of elevation of 30°. The ratio of the total time taken by the ball to hit the ground to its time of flight (time taken to come back to the same elevation) is (Take $g = 10ms^{-2}$)

(A) 2 : 1 (B) 3 : 1

(C) 3 : 2 (D) 1.5 : 1

14. In Q.13, the horizontal displacement (from the foot of the tower) of the ball is approximately equal to

(A) $50m$ (B) $60m$ (C) $70m$ (D) $80m$

15. A canon on a level plain is aimed at an angle α above the horizontal and with a muzzle velocity v_0 towards a vertical cliff a distance R away. Then the height from the bottom at which the shell strikes the side walls of the cliff is

(A) $R\sin\alpha - \frac{gR^2}{2v_0^2\sin^2\alpha}$ (B) $R\tan\alpha - \frac{gR^2}{2v_0^2\cos^2\alpha}$

(C) $R\cos\alpha - \frac{gR^2}{2v_0^2\cos^2\alpha}$ (D) $R\tan\alpha - \frac{gR^2}{2v_0^2\sin^2\alpha}$

16. It is possible to project a particle with a given velocity in two possible ways so as to make it pass through a point P at a distance r from the point of projection. The product of the times taken to reach this point in the two possible ways is then proportional to

(A) $\frac{1}{r}$ (B) r (C) r^3 (D) $\frac{1}{r^2}$

17. A projectile has a maximum range of $200m$. What is the maximum height attained by it?

(A) $25m$ (B) $50m$ (C) $75m$ (D) $100m$

18. A body thrown along a frictionless inclined plane of angle of inclination 30° covers a distance of $40m$ along the plane. If the body is projected with the same speed at an angle of 30° with the ground, it will have a range of (Take $g = 10ms^{-2}$)

(A) $20m$ (B) $20\sqrt{2}m$

(C) $20\sqrt{3}m$ (D) $40m$

19. A body is projected with the kinetic energy K at angle of 60° with the horizontal. Its kinetic energy at the highest point of its trajectory will be

(A) $2K$ (B) K (C) $\frac{K}{2}$ (D) $\frac{K}{4}$

20. A body, projected with a certain kinetic energy, has a horizontal range R. The kinetic energy will be minimum at a position of the projectile when its horizontal range is

(A) R (B) $\frac{3R}{4}$ (C) $\frac{R}{2}$ (D) $\frac{R}{4}$

21. The horizontal distance x and the vertical height y of a projection at a time t are given by

$$x = at \text{ and } y = bt^2 + ct$$

where a, b and c are constants. What is the magnitude of the velocity of the projectile, $1sec$ after it is fired?

(A) $\sqrt{a^2 + (2b+c)^2}$ (B) $\sqrt{2a^2 + (b+c)^2}$

(C) $\sqrt{2a^2 + (2b+c)^2}$ (D) $\sqrt{a^2 + (b+2c)^2}$

22. In Q. 21, the angle (θ) with the horizontal at which the projectile is given by

(A) $\theta = \tan^{-1}(\frac{a}{b})$ (B) $\theta = \tan^{-1}(\frac{b}{a})$

(C) $\theta = \tan^{-1}(\frac{a}{c})$ (D) $\theta = \tan^{-1}(\frac{c}{a})$

23. In Q. 21, the acceleration due to gravity is given by

(A) $-2a$ (B) $-2b$

(C) $-2c$ (D) $-ac + b$

24. In Q.21, the magnitude of the initial velocity of the projectile is given by

(A) $\sqrt{a^2 + b^2}$ (B) $\sqrt{b^2 + c^2}$

(C) $\sqrt{a^2 + c^2}$ (D) $\sqrt{b^2 - 4ac}$

25. Four projectiles are projected with the same speed at angles 20°, 35°, 60° and 75° with the horizontal. The range will be the longest for the projectile whose angle of projection is

(A) 20° (B) 35° (C) 60° (D) 75°

26. A player throws a ball which reaches the other player in $4sec$. If the height of each player is $1.8m$, what is the maximum height attained by the ball above the ground?

(A) $19.4m$ (B) $20.4m$

(C) $21.4m$ (D) $22.4m$

27. A projectile is thrown at an angle of 30° with the horizontal has a range R_1, and attains a maximum height h_1. Another projectile thrown, with the same velocity, at an angle 30° with the vertical, has a range R_2 and attains a maximum height h_2. The relation

between R_1 and R_2 is

(A) $R_1 = \frac{R_2}{2}$ (B) $R_1 = R_2$

(C) $R_1 = 2R_2$ (D) $R_1 = 4R_2$

28. In Q.27, what is the relation between h_1 and h_2?

(A) $h_1 = \frac{h_2}{4}$ (B) $h_1 = \frac{h_2}{3}$

(C) $h_1 = \frac{h_2}{2}$ (D) $h_1 = h_2$

29. The maximum height attained by a projectile is increased by 10% by increasing its speed of projection, without changing the angle of projection. The percentage increase in the

30. In Q.29, what is the percentage increase in the time of flight of the projectile?

(A) 20% (B) 15% (C) 10% (D) 5%

31. The maximum height attained by a projectile is increased by 10% by changing the angle of projection, without changing the speed of projection. The percentage increase in the time of flight will be

(A) 20% (B) 15% (C) 10% (D) 5%

32. The speed of projection of a projectile is increased by 5%, without changing the angle of projection. The percentage increase increase in the range will be

(A) 2.5% (B) 5% (C) 7.5% (D) 10%

33. In Q.32, what is the percentage increase in the maximum height attained by the projectile?

(A) 2.5% (B) 5% (C) 7.5% (D) 10%

34. In Q.32, what is the percentage increase in the time of fight of the projectile?

(A) 2.5% (B) 5% (C) 7.5% (D) 10%

35. A projectile has range R and time of flight T. If the range is doubled (by increasing the speed of projection, without changing the angle of projection), the time of flight will become

(A) $\frac{T}{\sqrt{2}}$ (B) $\sqrt{2}T$ (C) $\frac{T}{2}$ (D) $2T$

36. A projectile has the same range R when the maximum height attained by it is either h_1 or h_2. The R, h_1, and h_2 will be related as

(A) $R = \sqrt{h_1 h_2}$ (B) $R = 2\sqrt{h_1 h_2}$

(C) $R = 3\sqrt{h_1 h_2}$ (D) $R = 4\sqrt{h_1 h_2}$

37. A body is projected at time $t = 0$ from a certain point on a planet's surface with a certain velocity at a certain angle with the planet's surface. The horizontal and vertical displacements x and y respectively vary with time t (in seconds) as

$$x = 10\sqrt{3}t$$
$$y = 10t - t^2$$

What is the magnitude and direction of the velocity with which the body is projected?

(A) $20ms^{-1}$ at an angle 30° with the horizontal

(B) $20ms^{-1}$ at an angle 60° with the horizontal

(C) $10ms^{-1}$ at an angle 30° with the horizontal

(D) $20ms^{-1}$ at an angle 60° with the horizontal

38. What is the value of accelaration due to gracity on the planet's surface in Q.37?

(A) $1ms^{-2}$ (B) $2ms^{-2}$

(C) $4ms^{-2}$ (D) $9.8ms^{-2}$

39. What is the maximum height which the body in Q.37 will attain?

(A) $25m$ (B) $50m$

(C) $75m$ (D) $100m$

40. What is the total time of flight of the body in Q.37?

(A) $2s$ (B) $5s$ (C) $10s$ (D) $20s$

41. What is the distance between the point from where the body in Q.37 is projected and the point where it strikes the planet's surface?

(A) $25\sqrt{3}m$ (B) $50\sqrt{3}m$

(C) $75\sqrt{3}m$ (D) $100\sqrt{3}m$

42. A particle reaches its highest point when it has covered exactly one- half of its horizontal range. The corresponding point on the displacement- time graph is characterised by

(A) negative slope and zero curvature

(B) zero slope and negative curvature

(C) zero slope and positive curvature

(D) positive slope and zero curvature

43. A ball whose kinetic energy is E, is thrown at an angle of 45° with the horizontal. Its kinetic energy at the highest point of its trajectory will be

(A) E (B) $\frac{E}{\sqrt{2}}$ (C) $\frac{E}{2}$ (D) $zero$

44. An aeroplane is flying horizontally with a velocity of $600km/h$ at a height of $1960m$. When it is vertically above a point A on the ground, a bomb is released from it. The bomb strikes the point B. The distance AB is

(A) $1200m$ (B) $0.33km$

(C) $3.33km$ (D) $33km$

45. A ball is projected vertically upwards with a certain initial speed. Another ball of the same mass is projected at an angle of 60° with the vertical with the same initial speed. At the highest point, the ratio of their potential energies will be

(A) 4 : 1 (B) 3 : 2 (C) 2 : 3 (D) 2 : 1

46. A gun is located at the top of a hill of height h above the horizontal plane. It fires two shells with the same velocity u at angles α and β with the horizontal. If the shells strike the horizontal ground at the same point, show that

$$h = 2\frac{u^2(1-\tan\alpha\tan\beta)}{g(\tan\alpha+\tan\beta)^2}$$

47. Two stones are projected so as to reach the same distance from the point of projection on a horizontal surface. The maximum height reached by one exceeds the other by an amount equal to half the sum of the heights attained by them. Then the angles of projection for the stones are

(A) 45°, 135° (B) 0°, 90°

(C) 30°, 60° (D) 20°, 70°

48. A ball is projected with a speed v_0 at an angle α from a point on the playground. Then

(A) Its velocity is perpendicular to the initial velocity of projection at $t = \frac{v_0}{g}\sin\alpha$ for $0 < \alpha \leq \frac{\pi}{2}$

(B) Its velocity is perpendicular to the initial velocity of projection at $t = \frac{v_0}{g}\sin\alpha$ for $0 < \alpha \leq \frac{\pi}{4}$

(C) Its velocity is perpendicular to the initial velocity of projection at $t = \frac{v_0}{g}\sin\alpha$ for $\frac{\pi}{4} \leq \alpha \leq \frac{\pi}{2}$

(D) Its velocity is perpendicular to the initial velocity of projection at $t = \frac{v_0}{g}\sin\alpha$ for all values of α

49. A ball is projected so as to pass a wall at a distance a from the point of projection at an angle of 45° and falls at a distance b on the the other side of the wall. If h is the height of the wall then

(A) $h = a\sqrt{2}$ (B) $h = b\sqrt{2}$

(C) $h = \frac{\sqrt{2}ab}{a+b}$ (D) $h = \frac{ab}{a+b}$

50. A stone is thrown from a point at a distance a from a wall of height b. If it just clears the wall then the maximum height h reached by the stone for angle of projection α is

(A) $\frac{a^2\tan^2\alpha}{4(a\tan\alpha-b)}$ (B) $\frac{a^2\sec^2\alpha}{4(a\sec\alpha-b)}$

(C) $\frac{a^2\tan^2\alpha}{4b}$ (D) $\frac{a^2\tan^2\alpha}{4(a-b\cot\alpha)}$

51. A stone is projected so as to pass two walls of heights a and b at distances b and a respectively from the point of projection. If α is the angle of projection then

(A) minimum value of $\tan\alpha$ is $\sqrt{3}$

(B) minimum value of $\tan\alpha$ is 3

(C) maximum value of $\tan\alpha$ is $\sqrt{3}$

(D) maximum value of $\tan\alpha$ is 3

52. A stone is projected from a point on the ground so as to hit a bird on the top of a vertical pole of height h and then attain a maximum height $2h$ above the ground. If at the instant of projection the bird flies away horizontally with a uniform speed and if the stone hits the bird while descending then the ratio of the speed of the bird to the horizontal speed of

the stone is

(A) $\frac{\sqrt{2}}{\sqrt{2}+1}$ (B) $\frac{\sqrt{2}}{\sqrt{2}+2}$

(C) $\frac{1}{\sqrt{2}+1}$ (D) $\frac{2}{\sqrt{2}+1}$

53. The trajectory of a projectile in vertical plane is $y = ax - bx^2$, where a and b are constants and x and y are respectively horizontal and vertical distances of the projectile from the point of projection. The maximum height attained by the particle and the angle of projection from the horizontal are

(A) $\frac{b^2}{2a}$, $\tan^{-1}(b)$ (B) $\frac{a^2}{b}$, $\tan^{-1}(2a)$

(C) $\frac{a^2}{4b}$, $\tan^{-1}(a)$ (D) $\frac{2a^2}{b}$, $\tan^{-1}(a)$

54. The speed of a projectile when it is at its greatest height is $\frac{\sqrt{2}}{5}$ times its speed at half the maximum height. The angle of projection is

(A) $30°$ (B) $60°$

(C) $45°$ (D) $\tan^{-1}(\frac{3}{4})$

55. After one second the velocity of a projectile makes an angle of $45°$ with the horizontal. After another one second it is travelling horizontally. The magnitude of its initial velocity and angle of projection are (Take $g = 10ms^{-2}$)

(A) $14.62ms^{-1}$, $60°$

(B) $14.62ms^{-1}$, $\tan^{-1}(2)$

(C) $22.36ms^{-1}$, $\tan^{-1}(2)$

(D) $14.62ms^{-1}$, $60°$

56. A projectile is given a initial velocity of $\hat{i} + 2\hat{j}$. The Cartesian equation of its path is (Take $g = 10ms^{-2}$)

(A) $y = 2x - 5x^2$ (B) $y = x - 5x^2$

(C) $4y = 2x - 5x^2$ (D) $y = 2x - 25x^2$

57. At a height $0.4m$ from the ground , the velocity of a projectile in vector form is $\overrightarrow{v} = (6\hat{i} + 2\hat{j})ms^{-1}$ (the $x-$ axis is horizontal and $y-$ axis is vertically upwards). The angle of projection is (Take $g = 10ms^{-2}$)

(A) $45°$ (B) $60°$

(C) $30°$ (D) $\tan^{-1}\frac{3}{4}$

58. A hollow vertical cylinder of radius R and height h has smooth internal surface. A small particle is placed in contact with the inner side of the upper rim at a point P. It is given a horizontal speed v_0 tangential to rim. It leaves the lower rim at point Q, vertically below P. The number of revolutions made by the particle will be

(A) $\frac{h}{2\pi R}$ (B) $\frac{v_0}{\sqrt{2gh}}$

(C) $\frac{2\pi R}{h}$ (D) $\frac{v_0}{2\pi R\frac{\sqrt{2h}}{g}}$

59. The horizontal range and maximum height attained by a projectile are R and H respectively. If a constant horizontal acceleration $a = \frac{g}{4}$ is imparted to the projectile due to wind, then its horizontal range and maximum height will be

(A) $(R + H)$, $\frac{H}{2}$ (B) $(R + \frac{H}{2})$, $2H$

(C) $(R + 2H)$, H (D) $(R + H)$, H

60. A particle A is projected from the ground with an initial velocity of $10ms^{-1}$ at an angle of $60°$ with horizontal. From what height h should another particle B be projected horizontally with velocity $5ms^{-1}$ so that both particles collide in ground at point C if both are projected simultaneously (Take $g = 10ms^{-2}$)

(A) $10m$ (B) $30m$ (C) $15m$ (D) $25m$

61. Time taken by the projectile to reach from A to B is t. Then the distance AB is equal to

(A) $\frac{ut}{\sqrt{3}}$ (B) $\frac{\sqrt{3}ut}{2}$ (C) $\sqrt{3}ut$ (D) $2ut$

62. A particle is projected at angle of $60°$ above the horizontal with a speed of $10ms^{-1}$. After some time the direction of its velocity makes an angle of $30°$ above the horizontal. The speed of the particle at this instant is

(A) $\frac{5}{\sqrt{3}}ms^{-1}$ (B) $5\sqrt{3}ms^{-1}$

(C) $5ms^{-1}$ (D) $\frac{10}{\sqrt{3}}ms^{-1}$

63. With what minimum speed must a particle be projected from origin so that it is able to pass through a given point $(30m, 40m)$. (Take $g = 10ms^{-2}$)

(A) $60ms^{-1}$ (B) $30ms^{-1}$

(C) $50ms^{-1}$ (D) $40ms^{-1}$

64. Choose the correct alternative(s)

(A) If the greatest height to which a man can throw a stone is h, then the greatest horizontal distance upto which he can throw the stone $2h$

(B) The angle of projection for a projectile motion whose range R is n times the maximum height H is $\tan^{-1}(\frac{4}{n})$

(C) The time of flight T and the horizontal range R of a projectile are connected by the equation $gt^2 = 2R\tan\theta$ where θ is the angle of projection

(D) A ball is thrown vertically up. Another ball is thrown at an angle θ with the vertical. Both of them remain in air for the same period of time. Then the ratio of heights attained by the two balls is $1:1$

65. A ball is projected horizontally from the top of a building $19.6m$ high. How long will the ball take to hit the ground? If the line joining the point of projection to the point where it hits the ground is $45°$ with the horizontal, what must be the initial velocity of the ball? With what vertical velocity does the ball strike the ground? (Take $g = 9.8ms^{-2}$)

66. A bullet fired with a certain muzzle speed at an angle of $30°$ with the horizontal hits the ground $3km$ away. By adjusting its angle of projection, can the bullet be made to hit a target $5km$ away? Assume the muzzle speed to remain the same and neglect air resistance.

67. A gun kept on a straight horizontal road is used to hit a car travelling along the same speed of $72km/h$. The car is at distance of $500m$ from the gun when the gun is fired at an angle of $45°$ to the horizontal. Find

(i) the distance of the car from the gun when the shell hits it, and

(ii) the speed of projection of the shell from the gun.

68. An aeroplane is flying in a horizontal direction with a velocity of $600km/h$ and at a height of $1960m$. When it is vertically above the point A on the ground, a body is dropped from it. The body strikes the ground at point B. Calculate the distance AB.

69. A body falling freely from a given height H hits an inclined plane plane in its path at a height h. As a result of this impact, the direction of the velocity of the body becomes horizontal. For what value of $\frac{h}{H}$, will the body take maximum time to reach the ground?

70. Two guns situated at the top of a hill of height $10m$, fire one shot each with the same speed $5\sqrt{3}ms^{-1}$ at some interval of time. One gun fires horizontally and other fires upwards at an angle $60°$ with the horizontal. The shots collide in air at a point P. Find

(a) the time interval between the firings and

(b) the coordinates of point P.

Take the origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in the x-y plane.

71. An object A is kept at the point $x = 3m$ and $y = 1.25m$ on a plank P raised above the ground. At time $t = 0$ the plank starts moving along the $+x$ direction with an acceleration $1.5ms^{-2}$. At the same instant a stone is projected from the origin with a velocity u as shown in figure. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle $45°$ to the horizontal. All the motions are in $x-y$ plane. Find u and the time after which the stone hits the object. (Take $g = 10ms^{-2}$)

72. Two vertical walls, each of height h, are a distance $2h$ apart. A body is projected with a speed $2\sqrt{hg}$ at a certain angle θ with the horizontal such that it just clears the two walls. Show that time taken by the body to pass between the walls is $\Delta t = 2\sqrt{hg}$

73. Two balls are projected from a point at the same instant with velocities u and v. Those horizontal and vertical components are u_x, v_x and u_y, v_y respectively. Show that the time interval between their passing through the other common point on their trajectories is given by

$$\Delta t = 2d\frac{v_x u_y - v_y u_x}{g(u_x + v_x)}$$