Assignment P–11–6

(Particle Dynamics)

Questions

1. Why do you fall forward when a moving train decelerates to a stop and fall back when a train accelerates from rest? What would happen if the train rounded a curve at constant speed?

2. A horse is urged to pull a wagon. The horse refuses to try, citing Newton's third law as his defense: " 'The pull of the horse on the wagon is equal but opposite to the pull of the wagon on the horse.' If I can never exert a greater force on the wagon than it exerts on me, how can I ever start the wagon moving?" asks the horse. How would you reply?

3. A block of mass m is supported by a cord C from the ceiling, and another cord D is attached at the bottom of the block. Explain this: If you give a sudden jerk to D it will break, but if you pull on D steadily, C will break.

4. Using force, length, and time as fundamental quantities, what are the dimensions of mass?

5. In a tug of war, three men pull on a rope to the left at A and three men pull to the right at B with forces of equal magnitude. Now an object of mass 1 kg is hung vertically from the center of the rope AB. (a) Can the men get the rope AB to be horizontal? (b) If no, explain. If so, determine the magnitude of the forces at A and B to do this.

6. Both the following statements are true; explain them. Two teams having a tug of war must always pull equally hard on one another. The team that pushes harder against the ground wins.

7. Under what circumstances would your weight be zero? Does your answer depend on the choice of a reference frame?

8. Two objects of equal mass rest on opposite pans of beam balance. Does the scale remain balanced when it is accelerated up or down in an elevator?

9. A massless rope is strung over a frictionless pulley. A monkey holds onto one end of the rope and a mirror, having the same mass as the monkey, is attached to the other end of the rope at the monkey's level. Can the monkey get away from his image seen in the mirror (a) by climbing up the rope, (b) by climbing down the rope, (c) by releasing the rope?

10. A student standing on the large platform of a spring scale notes the reading. He then takes a step on this platform and notices that the scale reads less than the previous reading at the beginning of the step and more at the end of the step. Explain.

11. There is a limit beyond which further polishing of a surface *increases* rather than decreases frictional resistance. Can you explain this?

12. Is it unreasonable to expect a coefficient of friction to exceed unity?

13. How could a person who is at rest on completely frictionless ice covering a pond reach shore? Could he do this by walking, rolling, swinging his arms, or kicking his feet? How could a person be placed in such a position in the first place?

14. Explain how the range of a car's headlight limits the safe driving speed at night.

15. Your car skids across the center line on an icy highway. Should you turn the front wheels in the direction of skid or in the opposite direction (a) when you want to avoid a collision with an oncoming car, (b) when no other car is near but you want to regain control of the steering?

16. If you want to stop the car in the shortest distance on an icy road, should you (a) push hard on the brakes to lock the wheels, (b) push just hard enough to prevent slipping, or (c) "pump" the brakes?

17. A cube of mass m rests on a rough inclined plane making an angle θ with the horizontal. Compare the minimum force necessary to start the cube moving down the plane with that necessary to start the cube moving up the plane. How do these compare with the minimum *horizontal* force (tansverse to the slope) that will cause the cube to move down the plane?

18. Why are the train roadbeds and highways banked at curves?

19. How does the earth's rotation affect the apparent weight of a body at the equator?

20. A car is riding on a country road that resembles a roller coaster track. If the car travels with a constant speed, caompare the force it exerts on a horizontal section of the road to the force it exerts on the road at the top of a "hill" and at the bottom of a "valley". Explain

21. Suppose you need to measure whether a table top is a train is truly horizontal. If you use a spirit level, can you determine this when the train is moving down or up a slope? When the train is moving along a curve?

22. A coin is put on a phonograph turntable. The motor is started, but before the final speed of rotation is reached, the coin flies off. Explain.

23. A passenger in the front seat of a car finds himself sliding toward the door as the driver makes a sudden right turn. Describe the forces on the passenger and on the car at this instant if (a) the motion is viewed from a reference frame attached to the earth and (b) if attached to the car.

Problems

1. A block of mass M is pulled along a horizontal frictionless surface by means of a horizontal rope of mass m. A horizontal force F is applied to the free end of the rope. Find the force that the rope exerts on the block M.

2. Two blocks – A having mass 2.0 kg and B having mass 1.0 kg, are kept in contact on a horizontal frictionless surface. (a) If a horizontal force of 3.0 N is applied to A to push the two blocks, find the contact force between the blocks. (b) Show that if the same force is applied to B instead of A, the contact force between the two blocks is 2.0 N which is *not* the same as the value derived in (a). Explain.

3. A block slides down a frictionless incline making an angle θ with an elevator floor. Find its acceleration relative to the incline in the following cases. (a) Elevator descends at a constant speed v. (b) Elevator ascends at a constant speed v. (c) Elevator descends with constant acceleration a. (d) Elevator descends with constant deceleration a. (e) Elevator cable breaks.

4. A 4.0 kg block is put on the top of a 5.0 kg block resting on a smooth surface. In order to cause the top block to slip on the bottom one, a horizontal force of 12 N must be applied to the top block. Find (a) the maximum horizontal force F that can be applied to the lower block so that the two blocks will move together and (b) the resulting acceleration of the blocks.

5. A mass m is attached to the axis of a moving pulley (Figure 1). What force F should be applied to the end of the string passed around the second pulley for the mass to move *upwards* with an acceleration a? For the mass to be at rest? The pulleys and the string are massless and there is no friction.

6. Two masses m_1 and m_2 are connected by a string passing over a pulley. The surfaces on which they rest form angles α and β with the horizontal (Figure 2). The mass m_2 is lower than m_1 by a height h. Both masses will be at the same height in time τ after motion begins. The coefficients of friction between the masses and the surfaces are μ. Find the ratio m_1/m_2.

7. An inclined plane forms an angle $\alpha = 30°$ with the horizontal. A stone is thrown upwards along the plane and covers a distance $s = 16$ meters in $t_1 = 2$ seconds, after which it slides down. What time t_2 is required for the return journey? What is the coefficient of friction between the plane and the stone?

8. A cart with a mass of $M = 0.50$ kg is connected by a string to a mass $m = 0.20$ kg. At the initial moment the cart moves to the left along a horizontal plane at a speed $v_0 = 7$ m/s (Figure 3). Find the magnitude and the direction of the speed of the cart after $t = 5$ seconds and also the distance the cart has moved by this time.

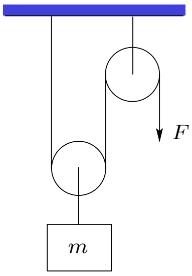

Figure 1: Problem 5

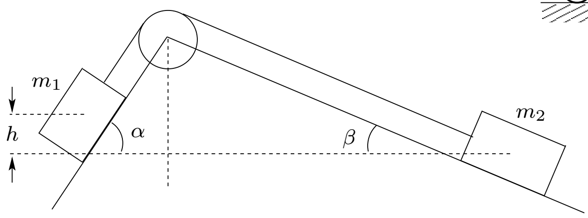

Figure 2: Problem 6

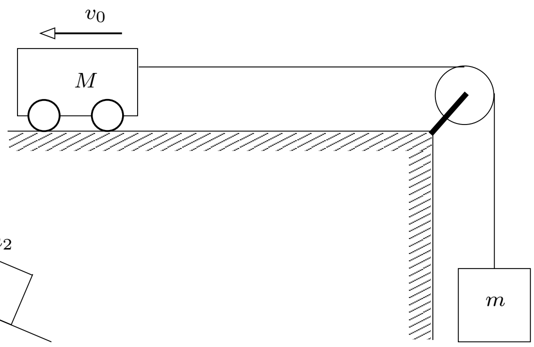

Figure 3: Problem 8

9. A bead of mass m is fitted on to a rod of length $2L$, and can move on it without friction. At the initial moment, the bead is in the middle of the rod. The rod moves translationally in a horizontal plane with an acceleration a in a direction forming an angle α with the rod (Figure 4). Find the acceleration of the bead relative to the rod, the reaction force by the rod on the bead, and the time taken by the bead to leave the rod.

10. Solve the previous problem, assuming that the moving bead is acted upon by a friction force, the coefficient of friction between the bead and the rod being μ. Disregard gravity.

11. A massless string thrown over a stationary pulley is passed through a hole (Figure 5). As the string moves, it is acted upon by a constant friction force f on the sides of the hole. The ends of the string carry masses m_1 and m_2. Find the acceleration of the masses.

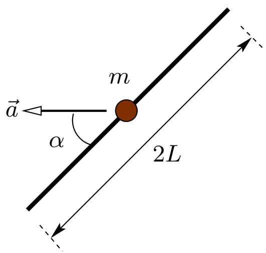

Figure 4: Problem 9

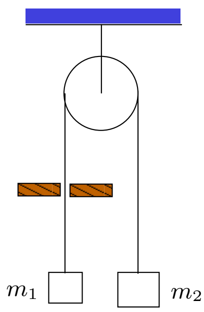

Figure 5: Problem 11

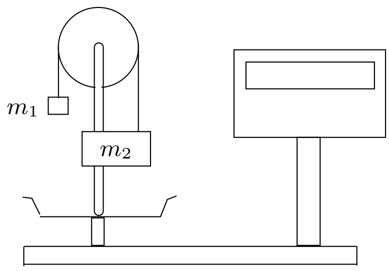

Figure 6: Problem 12

12. A stationary pulley is secured to the end of a light bar. The bar is placed onto a balance pan and secured in a vertical direction. Different masses are attached to the ends of a string (massless) passed over the pulley. One of the masses slides over the bar with some friction and therefore both masses move with constant speeds (Figure 6). Determine the force which the pulley acts on the bar with and the readings of the balance when the masses move. Disregard the masses of the pulley, bar, string and the friction in the pulley axis. Consider two cases: (i) $m_1 = 1$ kg, $m_2 = 3$ kg, and (ii) $m_1 = 3$ kg, $m_2 = 1$ kg.

13. In the Figure 7, masses m_1, m_2, and M are 20 kg, 5 kg, and 50 kg respectively. The coefficient of friction between M and the ground is zero. The coefficient of friction between m_1 and M and that between m_2 and the ground is $0 \cdot 3$. The pulleys and the strings are massless. The string is perfectly horizontal between P_1 and m_1 and also between P_2 and m_2. The string is perfectly vertical between

P_1 and P_2. An external horizontal force F is applied to the mass M. ($g = 10$ m/s^2)
(a) Draw a free body diagram of M, showing clearly all the forces.
(b) Let the magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and the ground be f_2. For a particular F, it is found that $f_1 = 2f_2$. Find f_1 and f_2. Write equations of motion of all the masses. Find F, tension in the string and accelerations of the masses.

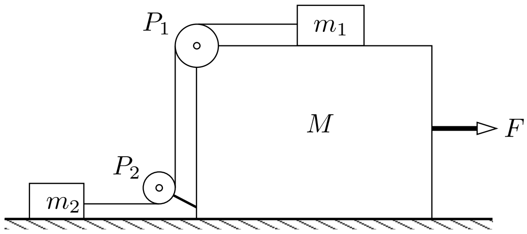

Figure 7: Problem 13

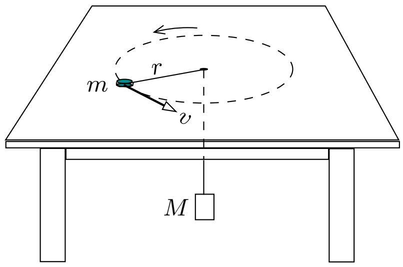

Figure 8: Problem 15

14. A homogenous chain of length ℓ lies on a table. What is the maximum length ℓ_1 of the part of the chain hanging over the table if the coefficient of friction between the chain and the table is μ?

15. A mass m on a horizontal table is attached to a hanging mass M by a masssless string through a hole in the table (Figure 8). Find the conditions (v and r) with which m must spin for M to stay at rest.

16. Imagine that the block m in Figure 8 is attached to a spring rather than a string. The unstretched length of the spring is ℓ_0 and the tension in the speing increases in direct proportion to its elongation, the tension per unit elongation being k. If the mass m rotates with a frequency f (number of revolutions per unit time), show that (a) the radius r of uniform circular motion is $\dfrac{k\ell_0}{k - 4\pi^2 m f^2}$ and (b) the tension T in the spring is $\dfrac{4\pi^2 mkf^2\ell_0}{k - 4\pi^2 m f^2}$. (c) Will the answer to part (a) and (b) change if the sense of rotation is opposite?

17. A particle is suspended vertically from a point O by an inextensible massless string of length ℓ. A vertical line AB is at a distance $\ell/8$ from O. The particle is given an initial speed u horizontally towards the line AB. At some point, its motion ceases to be circular and eventually the object passes throough the line AB. At the instant of crossing AB, its velocity is horizontal. Find u.

18. Two small blocks of mass $m_1 = 10$ kg and $m_2 = 5$ kg connected to each other by a massless inextensible string of length 0.3 m are placed along a diameter of a circular horizontal turntable. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular speed of 10 rad/s about a vertical axis through its center O. The masses are placed along the diameter of the table on either side of the center O such that m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turntable.

(i) Calculate the frictional force on m_1.

(ii) What should be the minimum angular speed of the turntable so that the masses will slip from their position.

(iii) How should the masses be placed with the string remaining taut so that there is no frictional force acting on m_1?

Answers to Problems

1. $\frac{M}{M+m}F$
2. (a) 1.0 N
3. (a) $g\sin\theta$, (b) $g\sin\theta$, (c) $(g-a)\sin\theta$, (d) $(g-a)\sin\theta$ (e) 0
4. (a) $F_{\max} = 15$ N, (b) $5/3$ m/s^2
5. $F = m(g+a)/2$ for the block to accelerate up; $F = mg/2$ for the block to be at rest
6. $\dfrac{m_1}{m_2} = \dfrac{g\tau^2(\sin\alpha+\sin\beta)(\sin\beta-\mu\cos\beta)+2h}{g\tau^2(\sin\alpha+\sin\beta)(\sin\alpha-\mu\cos\alpha)-2h}$
7. $t_2 = \dfrac{t_1}{\sqrt{\dfrac{gt_1^2}{s}\sin\alpha - 1}} = 4$ s, coefficient of friction $= \dfrac{1}{\cos\alpha}\left(\dfrac{2s}{gt_1^2} - \sin\alpha\right) = \dfrac{\sqrt{3}}{5} \approx 0.35$ taking $g = 10$ m/s^2
8. 7 m/s to the right; 17.5 m (assuming $g = 9.8$ m/s^2)
9. $a\cos\alpha$, $ma\sin\alpha$, $\sqrt{\dfrac{2L}{a\cos\alpha}}$
10. $a(\cos\alpha-\mu\sin\alpha)$, $ma\sin\alpha$, $\sqrt{\dfrac{2L}{a(\cos\alpha-\mu\sin\alpha)}}$
11. $a = \dfrac{(m_1-m_2)g - f}{m_1+m_2}$
12. The force with which the pulley acts on the bar is $2m_1g = 20$ N for case (i) and 60 N for case (ii). Reading of the balance = 40 N for both cases.
13. $f_1 = 30$ N, $f_2 = 15$ N, $F = 60$ N, $T = 18$ N, $a_{m_1} = a_{m_2} = a_M = 3/5$ m/s^2
14. $\ell_{1,\max} = \dfrac{\mu}{1+\mu}\ell$
15. $\dfrac{v^2}{r} = \dfrac{M}{m}g$
17. $u = \sqrt{\left(2+\dfrac{3\sqrt{3}}{2}\right)g\ell}$
18. (i) 36 N, (ii) $\omega_{\min} = \dfrac{25\sqrt{2}}{3}$ rad/s, (iii) $r_1 = 0.1$ m, $r_2 = 0.2$ m