Answers to Assignment P–11–09

1. Disc A with $\omega_A = 10.5$ rad/s
2. 0.105 rad/s, 1.75×10^{-3} rad/s, 7.27×10^{-5} rad/s
3. 2 s, $\pi/2$ rad/s
4. $\varphi_0 + \omega_0 t$, $\vec{v} = \omega_0(-y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$, $\vec{a} = -\omega_0^2(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})$
5. (i) (30 rad/s)$\hat{\mathbf{k}}$ (ii) 0.3 m
6. (i) 1.2 rad/s (ii) no (iii) no; direction of velocity vector and the centripetal acceleration
7. $v_A^2 \tan\theta/R^2 = 38.4$ rad/s^2
8. (i) $\vec{\omega} = -\dfrac{3v_G}{2r}\hat{\mathbf{k}} = -1.1 \times 10^1\ \hat{\mathbf{k}}$, (ii) point with highest linear speed is $(5r/3)\hat{\mathbf{i}} = \hat{\mathbf{i}}/3$ m with velocity $5\vec{v}_G/2 = -3.8\hat{\mathbf{j}}$ m/s
9. 2.6×10^2 N
10. $\dfrac{v}{R+r}\sin\alpha\,\tan(\alpha/2)$
11. $v_0 = \dfrac{v}{\sin\alpha - (r/R)}$ to the right
12. (i) -0.9 kN·m $\hat{\mathbf{k}}$, (ii) -0.2 kN·m $\hat{\mathbf{k}}$, (iii) -2 kN·m $\hat{\mathbf{k}}$
13. $\alpha = \dfrac{|m_2R - m_1r|}{m_2R^2 + m_1r^2 + I}g$; tension in the thread connected to m_1 is $T_1 = \dfrac{m_2R(R+r) + I}{m_2R^2 + m_1r^2 + I}m_1g$, tension in the thread connected to m_2 is $T_2 = \dfrac{m_1r(r+R) + I}{m_2R^2 + m_1r^2 + I}m_2g$
14. (i) $a_B = \dfrac{3g\sin\theta_0\cos\theta_0}{1 + 3\cos^2\theta_0}$ (ii) $v_A = \sqrt{3g\ell\sin\theta_0}$ **15.** $\sin\alpha = r/R$. For $Tr \le fR$ (f being the friction force), the spool will remain in place; otherwise, it will begin to rotate counter-clockwise about its center.
16. $v = \sqrt{\dfrac{(M+m)v_0^2 + \rho xgR}{M + m - \rho x}}$
17. $v_c = \omega_0 R/2$
18. If $v_0 < \omega_0 r$, the hoop will stop momentarily after time mv_0/f when rotating with angular speed $\omega = \omega_0 - \dfrac{v_0}{r}$. Then the hoop begins to move with slipping in the reverse direction. After some time the hoop will stop slipping and will roll without slipping to the left with $v_c = \dfrac{\omega_0 r - v_0}{2}$. If $v_0 > \omega_0 r$, then after time $m\omega_0 r/f$, the hoop will stop rotating momentarily and will move to the right with a translational velocity $v_0 - \omega_0 r$. Then the hoop begins to rotate in the reverse direction. After some time the hoop will stop slipping and will roll without slipping to the right with $v_c = \dfrac{v_0 - \omega_0 r}{2}$. If $v_0 = \omega_0 r$, the hoop will stop completely after time mv_0/f.
19. If at the moment when end B of the rod begins to rise, the force of friction $f \le \mu N$ is sufficient for end A not to slip, the rod will begin to rotate around point A. Otherwise, end A will begin to slip until the force of friction $f = \mu N$ can keep the rod in equilibrium. After this the rod will begin to rotate about end A. With $\mu = \cot\alpha$, slipping stops at an angle α which the thread BC forms with the rod. And for the rod not to slip at all, it is necessary that $\mu \ge \cot 60° = 1/\sqrt{3}$
20. $a_{board} = \dfrac{F}{M+m}$. Friction force at the contact with ground is zero while at the contacts of rollers with board, it is $f = \dfrac{m}{2(M+m)}F$
21. $v_0 \le \sqrt{8gR}$

Answers to Assignment P–11–09

1. Disc A with $\omega_A = 10.5$ rad/s
2. 0.105 rad/s, 1.75×10^{-3} rad/s, 7.27×10^{-5} rad/s
3. 2 s, $\pi/2$ rad/s
4. $\varphi_0 + \omega_0 t$, $\vec{v} = \omega_0(-y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$, $\vec{a} = -\omega_0^2(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})$
5. (i) (30 rad/s)$\hat{\mathbf{k}}$ (ii) 0.3 m
6. (i) 1.2 rad/s (ii) no (iii) no; direction of velocity vector and the centripetal acceleration
7. $v_A^2 \tan\theta/R^2 = 38.4$ rad/s^2
8. (i) $\vec{\omega} = -\dfrac{3v_G}{2r}\hat{\mathbf{k}} = -1.1 \times 10^1\ \hat{\mathbf{k}}$, (ii) point with highest linear speed is $(5r/3)\hat{\mathbf{i}} = \hat{\mathbf{i}}/3$ m with velocity $5\vec{v}_G/2 = -3.8\hat{\mathbf{j}}$ m/s
9. 2.6×10^2 N
10. $\dfrac{v}{R+r}\sin\alpha\,\tan(\alpha/2)$
11. $v_0 = \dfrac{v}{\sin\alpha - (r/R)}$ to the right
12. (i) -0.9 kN·m $\hat{\mathbf{k}}$, (ii) -0.2 kN·m $\hat{\mathbf{k}}$, (iii) -2 kN·m $\hat{\mathbf{k}}$
13. $\alpha = \dfrac{|m_2R - m_1r|}{m_2R^2 + m_1r^2 + I}g$; tension in the thread connected to m_1 is $T_1 = \dfrac{m_2R(R+r) + I}{m_2R^2 + m_1r^2 + I}m_1g$, tension in the thread connected to m_2 is $T_2 = \dfrac{m_1r(r+R) + I}{m_2R^2 + m_1r^2 + I}m_2g$
14. (i) $a_B = \dfrac{3g\sin\theta_0\cos\theta_0}{1 + 3\cos^2\theta_0}$ (ii) $v_A = \sqrt{3g\ell\sin\theta_0}$ **15.** $\sin\alpha = r/R$. For $Tr \le fR$ (f being the friction force), the spool will remain in place; otherwise, it will begin to rotate counter-clockwise about its center.
16. $v = \sqrt{\dfrac{(M+m)v_0^2 + \rho xgR}{M + m - \rho x}}$
17. $v_c = \omega_0 R/2$
18. If $v_0 < \omega_0 r$, the hoop will stop momentarily after time mv_0/f when rotating with angular speed $\omega = \omega_0 - \dfrac{v_0}{r}$. Then the hoop begins to move with slipping in the reverse direction. After some time the hoop will stop slipping and will roll without slipping to the left with $v_c = \dfrac{\omega_0 r - v_0}{2}$. If $v_0 > \omega_0 r$, then after time $m\omega_0 r/f$, the hoop will stop rotating momentarily and will move to the right with a translational velocity $v_0 - \omega_0 r$. Then the hoop begins to rotate in the reverse direction. After some time the hoop will stop slipping and will roll without slipping to the right with $v_c = \dfrac{v_0 - \omega_0 r}{2}$. If $v_0 = \omega_0 r$, the hoop will stop completely after time mv_0/f.
19. If at the moment when end B of the rod begins to rise, the force of friction $f \le \mu N$ is sufficient for end A not to slip, the rod will begin to rotate around point A. Otherwise, end A will begin to slip until the force of friction $f = \mu N$ can keep the rod in equilibrium. After this the rod will begin to rotate about end A. With $\mu = \cot\alpha$, slipping stops at an angle α which the thread BC forms with the rod. And for the rod not to slip at all, it is necessary that $\mu \ge \cot 60° = 1/\sqrt{3}$
20. $a_{board} = \dfrac{F}{M+m}$. Friction force at the contact with ground is zero while at the contacts of rollers with board, it is $f = \dfrac{m}{2(M+m)}F$
21. $v_0 \le \sqrt{8gR}$