Assignment P–11–09

(Rotational Mechanics)

1. Two discs A and B rotate at constant speeds about their centres. Disc A rotates at 100 rpm and disc B rotates at 10 rad/s. Which is rotating faster?

2. Find the angular speeds of the second, minute, and hour hands of a clock.

3. A disc rotates at 15 rpm. How many seconds does it take to rotate by 180°? What is the angular speed of the disc in rad/s?

4. A motor turns a uniform disc of radius R counter-clockwise about its mass center at a constant rate ω_0 The disc lies in the xy-plane and its angular displacement φ is measured (positive counter-clockwise) from the x-axis. What is the angular displacement $\varphi(t)$ of the disc if it starts at $\varphi(0) = \varphi_0$? What are the velocity vector and the acceleration vector of a point P at position $\vec{\mathbf{r}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$?

5. A 0.4 m long rod AB has many holes equally spaced along its length such that it can be pegged at any of the various locations. It rotates counter-clockwise at a constant angular speed about a peg whose location is not known. At some instant t, the velocity of end B is $\vec{\mathbf{v}}_B = -3\,\hat{\mathbf{j}}$ m/s. After $\pi/20$ s, the velocity of end B is $\vec{\mathbf{v}}_B = -3\,\hat{\mathbf{i}}$ m/s. If the rod has not completed one revolution during this period, (i) find the angular velocity of the rod, and (ii) find the location of the peg along the length of the rod from the end A.

6. A circular disc of radius $r = 250$ mm rotates in the xy-plane about a point which is at a distance $d = 2r$ away from the center of the disk. At the instant of interest, the linear speed of the center C is 0.60 m/s and the magnitude of its centripetal acceleration is 0.72 m/s^2.

(i) Find the rotational speed of the disk.

(ii) Is the given information enough to locate the center of rotation of the disk?

(iii) If the acceleration of the center has no component in the $\hat{\mathbf{j}}$ direction at the moment of interest, can you locate the center of rotation? If yes, is the point you locate unique? If not, what other information is required to make the point unique?

7. The circular disc of radius $R = 100$ mm rotates about its center O (see Fig. 1). At a given instant, point A on the disk has a velocity $v_A = 0.8$ m/s in the direction shown. At the same instant, the tangent of the angle θ made by the total acceleration vector of any point B with its radial line to O is 0.6. Compute the angular acceleration α of the disc.

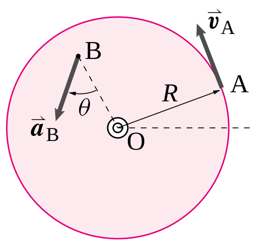

Figure 1: Problem 7.

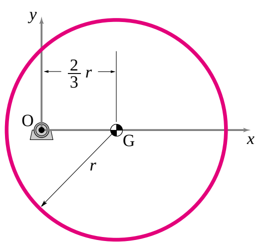

Figure 2: Problem 8.

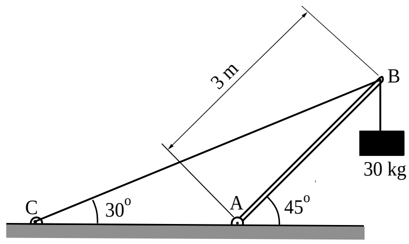

Figure 3: Problem 9.

8. A uniform disc of radius $r = 200$ mm is mounted eccentrically on a motor shaft at point O. The motor rotates the disc at a constant angular speed. At the instant shown, the velocity of the center of mass is $\vec{\mathbf{v}}_G = -1.5\,\hat{\mathbf{j}}$ m/s. (i) Find the angular velocity of the disc. (ii) Find the point with the highest linear speed on the disc. What is its velocity?

9. For static equilibrium of the system and the configuration shown in Fig. 3, find the support reaction at end A of the bar.

10. A bobbin rolls without slipping over a horizontal surface so that the velocity v of the end of the thread (point A) is directed along the horizontal. A board hinged at point B leans against the bobbin (see Fig. 4). The inner and outer radii of the bobbin are r and R. Determine the angular speed ω of the board as a function of the angle α.

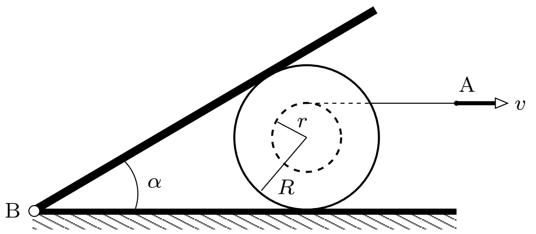

Figure 4: Problem 10.

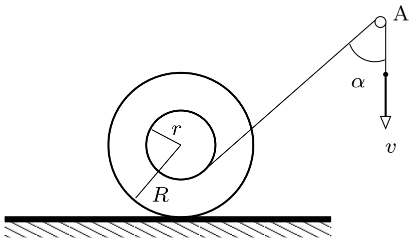

Figure 5: Problem 11.

11. The free end of a thread wound on a bobbin of inner radius r and outer radius R is passed around a nail A hammered into the wall (Fig. 5). The thread is pulled at a constant velocity v. Find the velocity v_0 of the centre of the bobbin at the instant when the thread forms an angle α with the vertical, assuming that the bobbin rolls over the horizontal surface without slipping.

12. Calculate the torque of the 2 kN payload on the robot arm about (i) joint A, (ii) joint B, and (iii) joint O if $\ell_1 = 0.8$ m, $\ell_2 = 0.4$ m, and $\ell_3 = 0.1$ m.

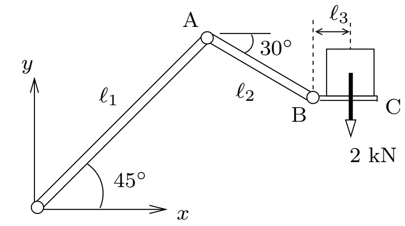

Figure 6: Problem 12.

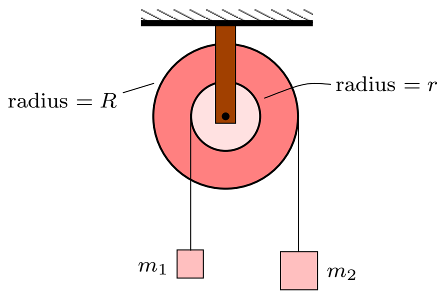

Figure 7: Problem 13.

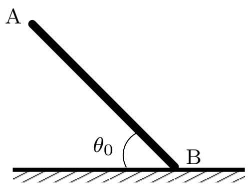

Figure 8: Problem 14.

13. The compound pulley system shown in Fig. 7 consists of two pulleys of radius r and R rigidly connnected to each other. The combined moment of inertia of the two pulleys about the axis of rotation is I. The two masses m_1 and m_2 are released from rest. Find the angular acceleration of the pulleys and the tensions in each string.

14. A homogeneous straight rod AB of mass m and length ℓ is held in the position shown in Fig. 8. The horizontal surface is frictionless. At some instant, the rod is released at rest. Find (i) the acceleration of point B just after the rod is released; (ii) the velocity of end A just before it hits the floor.

15. A heavy spool of inner radius r and outer radius R with a thread wound on it lies on a horizontal surface over which it can roll without slipping. If the thread is pulled to the left in a horizontal direction, the spool will also move to the left. If the direction of the thread is changed (see Fig. 9), the spool will begin to roll to the right at a certain angle α between the thread and the vertical. Determine this angle. How will the spool move if the thread is pulled at this angle.

16. A flexible cable is wound in one row around a hollow cylinderical drum of mass m and radius R. The mass of unit length of the cable is ρ while its total mass is M. Initially, the entire cable is wound on the drum and it lies on a horizontal floor on which it can roll without slipping. At some instant, the center of the cylinder acquires a velocity v_0 that is directed horizontally and perpendicular to its axis. Assume that the radius of the cable is negligible compared to that of the drum and determine the speed of the center of the cylinder at the instant when a length x of the cable lies on the surface.

17. A hoop of radius R rotating at an angular speed ω_0 is slowly lowered on a rough horizontal surface keeping the plane of the hoop perpendicular to the surface. Determine the speed of the center of the hoop when the hoop ceases to slip. At the initial moment the velocity of the center of the hoop is zero.

18. A hoop with radius r rotating with angular speed ω_0 is placed on a rough horizontal surface. A translational velocity v_0 is imparted to the hoop as shown in Fig 10. Determine the nature of the motion of the hoop, assuming that the force of sliding friction is f. Consider possible cases.

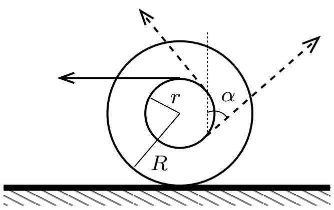

Figure 9: Problem 15.

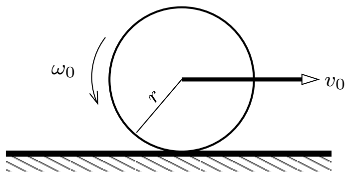

Figure 10: Problem 18.

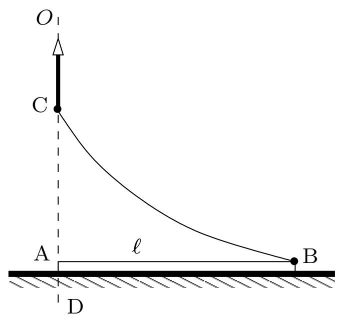

Figure 11: Problem 19.

19. A homogeneous thin rod AB with a length ℓ is placed on a rough horizontal surface of a table. A string with a length of 2ℓ is attached to end B of the rod (see Fig. 11). How will the rod move if the other end C of the string is slowly lifted up a stationary vertical straight line DO passing through end A of the rod. Disregard the weight of the string.

20. A wooden board with a mass M is placed on two identical hollow cylindrical rollers, each of mass m and radius R. The rollers rest on a horizontal surface. At the initial moment the system is at rest. Then, a force F is applied to the board in a horizontal direction. Find the acceleration of the board and the forces of friction acting between the rollers and the horizontal surface.

21. A point mass A is fixed to the inside of a thin rigid hoop of radius R and mass equal to that of A. The hoop rolls without slipping over a horizontal plane; at the moments when the mass A gets into the lower position, the center of the hoop moves with the velocity v_0. At what values of v_0 will the hoop move without bouncing?

22. A board of mass M, whose upper surface is rough and the under surface smooth, rests on a smooth horizontal plane. A sphere of mass m is placed on the board and the board is suddenly given a velocity v_0 in the direction of its length. If the coefficient of friction between the board and the sphere be μ, show that the sphere will begin to roll after a time $\dfrac{v_0}{\mu g\left(\dfrac{7}{2}+\dfrac{m}{M}\right)}$.

23. A cylinder rolls down a smooth plane whose inclination to the horizontal is α, unwrapping, as it goes, a fine string fixed to the highest point of the plane; find its acceleration and the tension of the string.

24. A circular cylinder, whose center of mass is at a distance c from its axis, rolls on a horizontal plane. If it be just started from a position of unstable equilibrium, show that the normal reaction of the plane when the center of mass is at its lowest position is $1 + \dfrac{4c^2}{(a-c)^2 + k^2}$ times its weight, where k is the radius of gyration about an axis through its center of mass.

25. A rough uniform rod, of length $2a$, is placed on a rough table at right angles to the table's edge; if its center of mass m be initially at a distance b beyond the edge, show that the rod will begin to slide when it has turned through an angle $\tan^{-1} \dfrac{\mu a^2}{a^2 + 9b^2}$, where μ is the coefficient of friction.

26. A solid homogeneous sphere, resting on top of another fixed sphere, is slightly displaced and begins to roll down it. Show that it will slip when the common normal makes with the vertical an angle θ given by the equation

$$2\sin(\theta - \lambda) = 5\sin\lambda(3\cos\theta - 2),$$

where λ is the angle of friction.

27. A solid homogeneous sphere is rolling on the inside of a fixed hollow sphere, the two centres being always in the same vertical plane. Show that the smaller sphere will make complete revolutions if, when it is in its lowest position, the reaction on it from the other sphere is greater than $\frac{34}{7}$ times its own weight.

28. A homogeneous sphere, of radius a, rotating with angular speed ω about a horizontal diameter, is gently placed on a table whose coefficient of friction is μ. Show that there will be slipping at the point of contact for a time $\dfrac{2\omega a}{7\mu g}$, and the then the sphere will roll with angular speed $\dfrac{2\omega}{7}$.

29. A uniform sphere, of radius a, is rotating about a horizontal diameter with angular speed ω and is gently placed on a rough plane which is inclined to the horizontal at an angle α, the sense of rotation being such as to tend to cause the sphere to move up the incline along the line of greatest slope. Show that, if the coefficient of friction be $\tan\alpha$, the center of the sphere will remain at rest for a time $\dfrac{2a\omega}{5g\sin\alpha}$, and will then move downwards with acceleration $\dfrac{5}{7}g\sin\alpha$.

30. A homogeneous sphere, of mass M, is placed on an imperfectly rough table, and a particle, of mass m, is attached to the end of a horizontal diameter. Show that the sphere will begin to roll or slide according as the coefficient of friction μ is greater or less than $\dfrac{5(M+m)m}{7M^2 + 17Mm + 5m^2}$. If μ be equal to this value, show that the sphere will begin to roll.

31. If a uniform semi-circular wire be placed in a vertical plane with one extremity on a rough horizontal plane, and the diameter through that extremity vertical, show that the semi-circle will begin to roll or slide according as the coefficient of friction $\mu \gtrless \dfrac{\pi}{\pi^2 - 2}$. If μ has this value, prove that the wire will roll.

32. A uniform stick, of length $2a$, hangs freely from one end, the other being close to the ground slightly above it. An angular speed ω is then given to the stick, and when it has turned through a right angle the fixed end is let go. Show that on first touching the ground it will be in an upright position if

$$\omega^2 = \frac{g}{2a}\left(3 + \frac{p^2}{p+1}\right)$$

where p is any odd multiple of $\pi/2$.