Assignment

(Plane motion. Projectile)

1. Suppose that the position vector for a particle is given as $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ with $x = At + B$ and $y = Ct^2 + D$, where $A = 1.00$ m/s, $B = 1.00$ m, $C = 0.125$ m/s^2 and $D = 1.00$ m. (a) Calculate the average velocity during the time interval from $t = 2.00$ s to $t = 4.00$ s. (b) Determine the velocity and the speed at $t = 2.00$ s.

2. A golf ball is hit off a tee at the edge of a cliff. Its x and y coordinates versus time are given by the following expressions:

$$x = (18.0 \text{ m/s})t, \qquad \text{and} \qquad y = (4.00 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$$

(a) Write a vector expression for the ball's position as a function of time, using the unit vectors $\vec{\mathbf{i}}$ and $\vec{\mathbf{j}}$. By taking derivatives of your results, write expressions for (b) the velocity vector as a function of time and (c) the acceleration vector as a function of time. Now use unit vector notation to write expressions for (d) the position, (e) the velocity, and (f) the acceleration of the ball, all at $t = 3.00$ s.

3. A particle initially located at the origin has an acceleration of $\vec{\mathbf{a}} = 3.00\hat{\mathbf{j}}$ m/s^2 and an initial velocity of $\vec{\mathbf{v}}_0 = 5.00\hat{\mathbf{i}}$ m/s. Find (a) the position vector and velocity at any time t and (b) the coordinates and speed of the particle at $t = 2.00$ s.

4. An astronaut on a strange planet finds that she can jump a maximum horizontal distance of 15.0 m if her initial speed is 3.00 m/s. What is the free–fall acceleration on the planet?

5. A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection? Give your answer to three significant figures.

6. A firefighter a distance d from a burning building directs a stream of water from a hosepipe at angle α above the horizontal. If the initial speed of the stream is v_0, at what height h does the water strike the building?

7. A stone is thrown horizontally with a speed $v_0 = 15$ m/s from a tower of height $h = 25$ m. Find: (a) the time for which the stone moves before striking the ground, (b) the distance s between the tower base and the point on the ground where the stone strikes, (c) the speed v with which the stone touches the ground, (d) the angle φ formed by the trajectory of the stone with the horizontal at the point where it reaches the ground.

8. A ball is thrown with some initial speed at a certain angle with the horizontal. The horizontal range of the ball is R, and the ball reaches a maximum height $R/6$. In terms of R and g, find (a) the time the ball is in motion, (b) the ball's speed at the peak of its path, (c) the initial vertical component of its velocity, (d) its initial speed, and (e) the angle of projection. (f) Suppose the ball is thrown at the same initial speed found in part (d) above but at the angle appropriate for reaching the maximum height. Find this height. (g) Suppose the ball is thrown at the same initial speed but at the angle necessary for maximum range. Find this range.

9. An astronaut standing on the Moon fires a gun so that the bullet leaves the barrel initially moving in a horizontal direction. (a) What must be the muzzle speed of the bullet so that it travels completely around the Moon and returns to its original location? (b) How long does this trip around the Moon take? Assume that the free–fall acceleration on the Moon is one–sixth that on the Earth.

10. A boy can throw a ball a maximum horizontal distance of R on a level field. How far can he throw the same ball vertically upward? Assume that his muscles give the ball the same speed in each case.

11. A ball starts falling with zero initial speed on a smooth inclined plane forming an angle α with the horizontal. Having fallen the distance h, the ball rebounds elastically off the inclined plane. At what distance from the impact point will the ball rebound for the second time?

12. A cannon and a target are 5.10 km apart and located at the same level. How soon will the shell launched with the initial speed of 240 m/s reach the target?

13. A cannon fires successively two shells with speed $v_0 = 250$ m/s; the first one at an angle $\alpha_1 = 60°$ and the second at an angle $\alpha_2 = 45°$ to the horizontal. Find the time interval between the two fires that will lead to the collision of the shells. The two shells move in the same plane.

14. A body is thrown from the surface of the Earth with an initial speed v_0. Draw the approximate graph of the tangential and normal components of the acceleration: a_n and a_τ of the projectile as a function of time.

15. A ball is to be thrown from a point on the ground over a vertical wall of height h that is a distance d away from the point of projection. At what minimum initial speed is this possible? At what angle α should the velocity be directed in this case?

16. Two stones are thrown at the same time and with the same initial speed v_0 from the origin at angles α_1 and α_2. What will be distance between the two stones after a time τ after the launch?

17. A passenger car is driving over a level highway bwhind a truck. A stone got stuck between the double tyres of the rear wheels of the truck. At what distance should the car follow the truck so that the stone will not strike it if it flies out from between the tyres. Both vehicles have a speed of 50 km/h.

18. A small ball thrown at an initial velocity v_0 at an angle α to the horizontal strikes a vertical smooth wall moving towards it at a horizontal velocity v and is bounced to the point from which it was thrown. Determine the time t from the beginning of motion to the moment of impact with the wall.

Answers

1. (a) $(1.00\,\hat{\mathbf{i}} + 0.750\,\hat{\mathbf{j}})$ m/s
(b) $(1.00\,\hat{\mathbf{i}} + 0.500\,\hat{\mathbf{j}})$ m/s, 1.12 m/s

2. (a) $(18.0t)\,\hat{\mathbf{i}} + (4.00t - 4.90t^2)\,\hat{\mathbf{j}}$
(b) $18.0\,\hat{\mathbf{i}} + (4.00 - 9.80t)\,\hat{\mathbf{j}}$
(c) $-9.80\,\hat{\mathbf{j}}$
(d) $(54.0\,\hat{\mathbf{i}} - 32.1\,\hat{\mathbf{j}})$ m
(e) $(18.0\,\hat{\mathbf{i}} - 25.4\,\hat{\mathbf{j}})$ m/s
(f) $(-9.80\,\hat{\mathbf{j}})$ m/s^2

3. (a) $\vec{\mathbf{r}} = (5.00\text{ m/s})t\,\hat{\mathbf{i}} + (1.50\text{ m/s}^2)t^2\,\hat{\mathbf{j}}$
$\vec{\mathbf{v}} = (5.00\text{ m/s})\,\hat{\mathbf{i}} + (3.00\text{ m/s}^2)t\,\hat{\mathbf{j}}$
(b) $x = 10.0$ m, $y = 6.00$ m, $v = 7.81$ m/s

4. 0.600 m/s^2

5. 53.1°

6. $d\tan\alpha - gd^2/(2v_0^2\cos^2\alpha)$

7. (a) 2.3 s (b) 34 m (c) 27 m/s (d) 56°

8. (a) $\sqrt{4R/3g}$ (b) $\frac{1}{2}\sqrt{3gR}$ (c) $\sqrt{gR/3}$
(d) $\sqrt{13gR/12}$ (e) 33.7° (f) $13R/24$
(g) $13R/12$

9. (a) $v_0 = \sqrt{R_m g_m} \approx 1.7$ km/s where R_m is the radius of the moon and g_m is the moon's acceleration due to gravity (b) 1.8 hr (approx)

10. $R/2$

11. $8h\sin\alpha$

12. 0.41 or 0.71 min depending on the projection angle

13. $\dfrac{2v_0}{g}\,\dfrac{\sin(\alpha_1 - \alpha_2)}{\cos\alpha_1 + \cos\alpha_2} = 11$ s

15. $v_0 = \sqrt{g\big(h + \sqrt{h^2 + d^2}\big)}$,
$$\alpha = \tan^{-1}\frac{d}{\sqrt{h^2 + d^2} - d}$$

16. $2v_0\tau\sin\left(\dfrac{\alpha_1 - \alpha_2}{2}\right)$

17. 19.6 m

18. $t = \dfrac{v_0\sin\alpha(v_0\cos\alpha + 2v)}{g(v_0\cos\alpha + v)}$