Assignment P–11–3

(Relative motion)

1. A train is travelling along a horizontal rail at the rate of 60 km/h. Rain is driven by the wind that is blowing in the same direction as the motion of the train, so that it falls at an angle of 30° with the vertical with a speed of 8.5 m/s. At what angle with the vertical does the rain seem to fall as seen by a person inside the train?

2. A ship is sailing due east with a speed of 12 km/h and an other ship is sailing due north at a speed of 16 km/h. What is the velocity of the second ship relative to the first one?

3. A ship is sailing north–east with a speed of 10 km/h. As seen by a passenger on board, the wind blows from the north with a speed of $10\sqrt{2}$ km/h. Find the wind velocity.

4. To a man walking at the speed of 2 km/h the rain seems to fall vertically; when he increases his speed to 4 km/h, it appears to meet him at an angle of 45°; find the velocity of rain.

5. One ship, sailing east with a speed of 15 km/h, passes a certain point exactly at noon; and a second ship, sailing north at the same speed, passes the same point at 1 : 30 PM. Find the minimum separation between them and the time at that moment.

6. An elevator car is moving up with constant acceleration. An object is thrown up in an elevator with a speed u relative to the elevator and the time during which it remains in free–fall is found to be τ. Find the acceleration of the elevator.

7. A ship sailing south–east sees another ship, which is moving at the same speed as itself, and which always appears to be in a direction due east and to be always coming nearer. Find the direction of motion relative to the Earth.

8. At a given moment of time, the distance between two particles, moving at constant velocities, is d. At this moment, let $\vec{v}$ be their relative velocity. If $\vec{v}_{||}$ be the component of $\vec{v}$ along the line joining the two particles and $\vec{v}_{\perp}$ be the component perpendicular to that line. Show that their separation when they are nearest to each other is $\dfrac{|\vec{v}_{\perp}|}{|\vec{v}|}\, d$ and that the time that elapses before they arrive at their nearest distance is $\dfrac{|\vec{v}_{||}|}{|\vec{v}|^2}\, d$.

9. To find the speed of an airplane it is necessary to determine how long it takes it to fly around a closed loop of a known length. How long will it take a plane to fly around a square with side ℓ, with the wind blowing at a speed u, in the following two cases: (i) the direction of the wind coincides with one of the sides of the square; and (ii) the direction of the wind coincides with a diagonal of the square? With no wind the speed of the plane is v which is greater than u.

10. Two motor vehicles run at constant speeds v_1 and v_2 along highways intersecting at an angle α. Find the magnitude and direction of the speed of one vehicle with respect to the other. In what time after they meet will the distance between the vehicles be s?

11. Two intersecting lines move translationally in opposite directions with speeds v_1 and v_2 perpendicular to the corresponding lines. The angle between the lines is α. Find the speed of the point of intersection of these lines.

12. Particle A moves uniformly with speed v so that the vector $\vec{v}$ is continually "aimed" at the particle B which in turn moves along a straight line with a constant speed u that is less than v. At the initial moment of time, $\vec{v} \perp \vec{u}$ and the particles are separated by a distance ℓ. How soon will the particles converge?

13. Smugglers set off in a ship in a direction perpendicular to a straight shore and move with a constant speed v. The coastguard's cutter is a distance a from the smugglers' ship and leaves the shore at the same time. The cutter always moves at constant speed in the direction of the smugglers' ship and catches up with the criminals when at a distance a from the shore. How many times greater is the speed of the coastguard's cutter than that of the smugglers' ship?

14. A train of length $\ell = 350$ m starts moving along a straight rail with constant acceleration $a_0 = 3.0 \times 10^{-2}$ m/s^2. After time $t_1 = 30$ s of the start of the motion, the locomotive headlight is switched on (event 1), and $t_2 = 60$ s after that event the tail signal light is switched on (event 2). Find the distance d between these events in the reference frame fixed to the train and the Earth. How and at what constant speed v relative to the Earth must a certain reference frame K move for the two events to occur in it at the same point?

Answers to Assignment P–11–3

1. 60°
2. 20 km/h at an angle $\tan^{-1}\frac{3}{4} = 37°$ west of north
3. 10 km/h along south–east
4. $2\sqrt{2}$ km/h at 45°
5. $\frac{45}{4}\sqrt{2}$ km at 12 : 45 PM
6. $(2u - g\tau)/\tau$
7. South–west
9. (i) $\dfrac{2\ell}{v}\dfrac{1+\sqrt{1-\beta^2}}{1-\beta^2}$, (ii) $\dfrac{4\ell}{v}\dfrac{\sqrt{1-\beta^2/2}}{1-\beta^2}$ where $\beta = \dfrac{u}{v}$
10. speed of 1st vehicle relative to the second (v_{12}) = speed of the 2nd vehicle relative to the 1st (v_{21}) $= \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos\alpha}$, $t = s/v_{12}$
11. $\dfrac{1}{\sin\alpha}\sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos\alpha}$
12. required time $\tau = v\ell/(v^2 - u^2)$
13. $\frac{1+\sqrt{5}}{2}$
14. $d = \ell - a_0t_2(t_1 + t_2/2) = 0.24$ km. The speed of K should be oppositely directed as that of the train with $v = 4.0$ m/s