

Assignment on Rational and Irrational Numbers

1. Let x and y be rational and irrational numbers, respectively. Is $x + y$ necessarily an irrational number? Give an example in support of your answer.

2. Let x and y be rational and irrational numbers, respectively. Is xy necessarily an irrational number? Give an example in support of your answer.

3. State whether the following statements are true or false. Justify your answer.

- (i) $\frac{\sqrt{2}}{3}$ is a rational number.
- (ii) There are infinitely many integers between any two integers.
- (iii) Number of rational numbers between 15 and 18 is finite.
- (iv) The square of an irrational number is always rational.
- (v) $\frac{\sqrt{12}}{\sqrt{3}}$ is not a rational number as $\sqrt{12}$ and $\sqrt{3}$ are not integers.

4. Classify the following numbers as rational or irrational with justification: (i) $\sqrt{196}$, (ii) $3\sqrt{18}$, (iii) 0.5918, (iv) $(1 + \sqrt{5}) - (4 + \sqrt{5})$, (v) 10.124124124... (vi) 1.01001000100001...

5. Locate on number line: (i) 7.2, (ii) $\frac{6}{5}$, (iii) $-\frac{12}{7}$, (iv) $\sqrt{13}$, (v) $\sqrt{10}$, (vi) $\sqrt{17}$

6. Express $0.12\bar{3}$ in the form $\frac{p}{q}$, where p and q are integers with $q \neq 0$.

7. Find which of the variables x , y , z and u represent rational numbers and which irrational numbers: (i) $x^2 = 5$, (ii) $y^2 = 9$, (iii) $z^2 = 0.04$ (iv) $u^2 = \frac{17}{4}$

8. Find four rational numbers between (i) 1 and -2 , (ii) 0.1 and 0.11 (iii) $\frac{5}{7}$ and $\frac{6}{7}$ (iv) $\frac{1}{4}$ and $\frac{1}{5}$.

9. Insert a rational and an irrational number between the following: (i) 2 and 3 (ii) 0 and 0.1 (iii) $\sqrt{2}$ and $\sqrt{3}$ (iv) 2.375 and 3.121 (vi) 6.375289 and 6.375738.

10. Show that $0.142857142857142857\dots = \frac{1}{7}$

11. Rationalize the following:

$$(i) \frac{2 + \sqrt{3}}{2 - \sqrt{3}}$$

$$(ii) \frac{3\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$$

$$(iii) \frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{48} + \sqrt{18}}$$

12. If $\frac{5 + 2\sqrt{3}}{7 + \sqrt{3}} = a - \sqrt{3b}$, find a and b , where a and b are rational numbers.

13. Simplify:

$$\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6} + \sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15} + 3\sqrt{2}}$$

14. Arrange the following in descending order of magnitude: $\sqrt[8]{90}$, $\sqrt[4]{10}$, $\sqrt{6}$.

15. If $r = \frac{3 + \sqrt{5}}{2}$, then find the value of $a^2 + \frac{1}{a^2}$.

16. Prove that the following numbers are irrational.

$$(i) \sqrt[3]{4} \quad (ii) \sqrt{6} \quad (iii) \frac{\sqrt{2}}{3} + 5 \quad (iv) \sqrt{2} + \sqrt{3}$$

17. If $x = \frac{\sqrt{2} + 1}{\sqrt{2} - 1}$ and $y = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$, find the value of $x^2 + y^2 + xy$.

18. Let r be a real number such that $\sqrt[3]{r} + \frac{1}{\sqrt[3]{r}} = 3$.

Determine the value of $r^3 + \frac{1}{r^3}$.

19. If $\sqrt{5 + \sqrt{5}} - \sqrt{5 - \sqrt{5}} = \sqrt{a + b\sqrt{5}}$, where a and b are integers, find a and b .

20. Find the value of

$$\sqrt{1 + 2\sqrt{1 + 2\sqrt{1 + 2\sqrt{1 + 2\sqrt{1 + \dots}}}}}$$

21. For a natural number n , define $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$, and additionally $0! = 1$. For example $5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$. Consider the number

$$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$$

Prove that e is irrational.