# Assignment M–11–4

(Permutations and combinations)

**1.** Out of 7 boys and 4 girls, a committee of 6 is to be formed; in how many ways can this be done, (i) when the committee contains exactly 2 girls, (ii) at least 2 girls?

**2.** Out of 7 consonants and 4 vowels, how many words can be formed each containing 3 consonants and 2 vowels? The words need not have any meaning.

**3.** In how many ways can you take an odd number of objects from $n$ objects?

**4.** How many ways can you split 14 people into 7 pairs?

**5.** There are $n$ boys and $n$ girls in a dance class. How many ways are there to pair them all up?

**6.** One student has 6 books and another has 8. In how many ways can they exchange 3 books of the first student for 3 books of the second?

**7.** A group of soldiers contains 3 officers, 6 sergeants, and 30 privates. How many ways can a team be formed consisting of 1 officer, 2 sergeants, and 20 privates?

**8.** A person has 10 friends. Over several days he invites some of them to a dinner party in such a way that he never invites exactly the same group of people. How many days can he keep this up, assuming that one of the possibilities is to ask nobody to dinner?

**9.** Each of the faces of a cube is colored by a different color. How many of the colorings are distinct?

**10.** In how many ways can $n$ things be given to $p$ persons, when there is no restriction as to the number of things he may receive?

**11.** In how many ways can 7 men and 7 women sit down at a round table, no two women together?

**12.** Find the number of ways of dividing $mn$ things into $n$ equal groups.

**13.** Find the number of permutations which can be formed out of the letters of the word *series* taken three together.

**14.** In how many ways can 5 horses finish a race.

**15.** There are $n$ different books and $p$ copies of each. Find the number of ways in which a selection can be made from them.

**16.** Find the number of selections and arrangements that can be made by taking 4 letters from the word *expression*.

**17.** Let $R_n$ be the number of ways of placing $n$ identical rooks on an $n \times n$ chessboard so that they do not attack each other. Find a formula for $R_n$. Further, let $M_n$ and $D_n$ be the number of those placing which remain unchanged upon reflections about a diagonal and about both diagonals respectively. Prove that $M_n = M_{n-1} + (n-1)M_{n-2}$, and $D_{2n} = 2D_{2n-2} + (2n-2)D_{2n-4}$, $D_{2n+1} = D_{2n}$ for integers $n$.

**18.** If of $p+q+r$ things, $p$ be alike of one kind, $q$ be alike of different kind and rest all be different, find the total number of selections that can be made.

**19.** $2n$ objects of each of three kinds are given to two persons, so that each person gets $3n$ objects. Prove that this can be done in $3n^2 + 3n + 1$ ways.

**20.** If the $n+1$ numbers $a_0$, $a_1$, $a_2$, $a_3$, …, $a_n$ be all different, and each of them a prime number, find the number of different factors of the expression $a_0^m a_1 a_2 a_3 \cdots a_n$.

**21.** Of $3n+1$ objects, $n$ are indistinguishable, and the remaining ones are distinct. Show that one can choose from them $n$ objects in $2^{2n}$ ways.

**22.** Find the sum $S_n = \sum_{k=1}^{n} {}^nC_k\, k^3$. Hint: the sum can be interpreted as the number of ways to choose a committee of at least one that includes a president, vice-president, and treasurer (not necessarily distinct persons) from a set of $n$ people.

**23.** Ten places in a row are each to be filled either with a 0, a 1, or a 2. How many ways can this be done if no two or more neighboring 0's should appear?

**24.** Find the highest power of 6 that is contained in 100!.

**25.** Six papers are set in an examination, two of them in mathematics. In how many different orders can the papers be given if the two mathematics paper are not successive?

**26.** We draw all diagonals of a convex $n$-sided polygon. Suppose no three diagonals pass through a point. Into how many regions is the interior of the polygon divided?

**27.** It is a rule in Gaelic that no consonant or group of consonants can stand immediately between a strong and a weak vowel; the strong vowel being *a, o, u* and the weak vowels *e* and *i*. Show that the total number of Gaelic words of $n+3$ letters each, each of which can be formed of $n$ consonants and the vowels *a, e, o*, where no letter is repeated in the same word is $2\dfrac{(n+3)!}{n+2}$.

**28.** $2n$ points are given on a circle. In how many ways can you join pairs of points by non-intersecting chords?

**29.** We have 20 presents that we want to distribute to 12 children. It is not necessary that every child gets something; in fact we might end up giving all the presents to a single child. In how many ways can we distribute the presents?

**30.** Suppose we have $n$ different presents, which we want to distribute to $k$ children. Let's call the children as 1, 2, 3, $\ldots k$ (not a very friendly way to name them!). We want that child 1 should get exactly $n_1$ presents, child 2 should get exactly $n_2$ presents, and so on and finally the last child gets $n_k$ presents. Obviously, we must have $n_1 + n_2 + n_3 + \ldots + n_k = n$. How many ways can we distribute the presents?